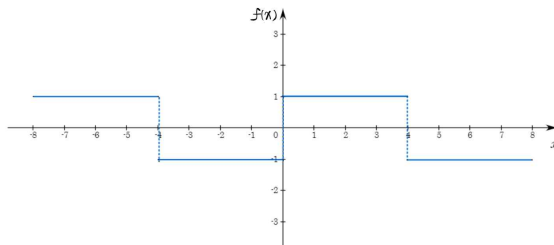


[Chapter 11] 연습문제 정답

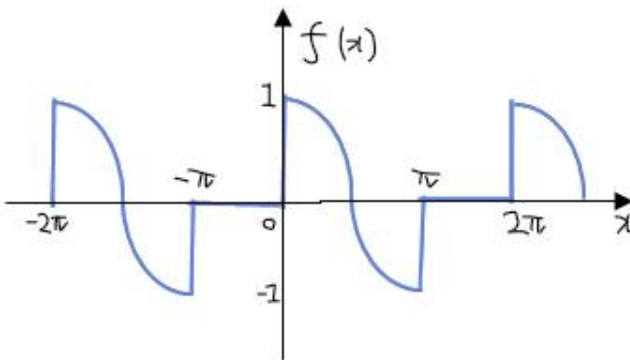
11.1

[풀이]



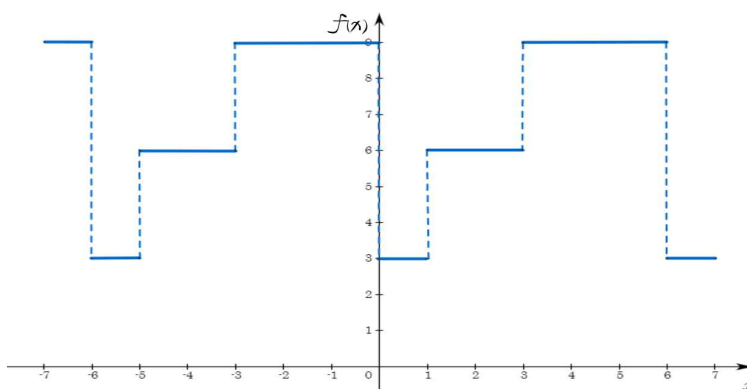
11.2

[풀이]



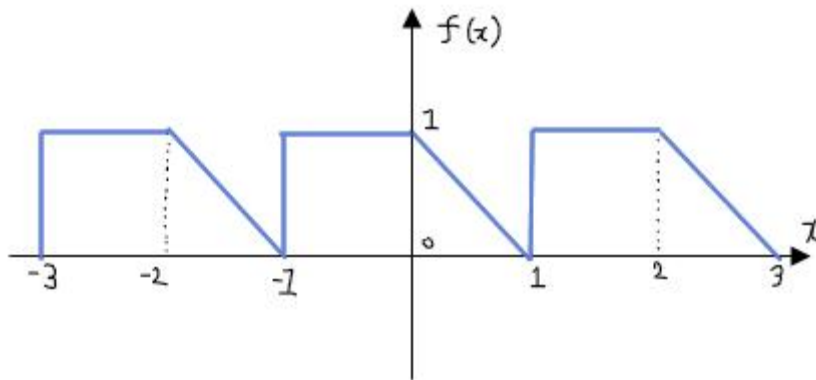
11.3

[풀이]



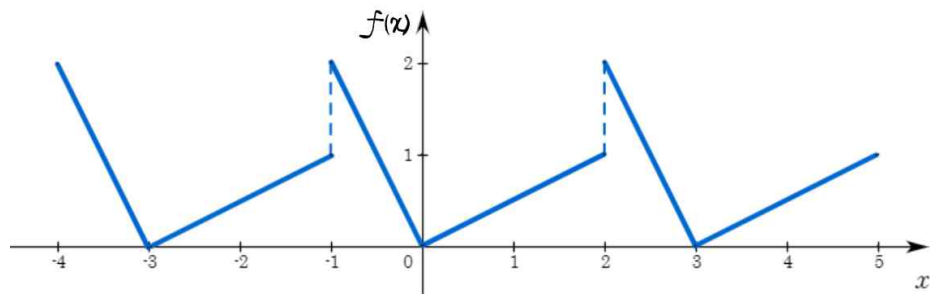
11.4

[풀이]



11.5

[풀이]



11.6

[풀이]

증명생략

11.7

[풀이]

증명생략

11.8

[풀이]

증명생략

11.9

[풀이]

증명생략

11.10

[풀이]

$$\therefore f(x) = \sum_{n=1}^{\infty} \left\{ \frac{4}{n} \left(\sin \frac{n\pi}{2} \right) (\cos nx) \right\}$$

11.11

[풀이]

$$\therefore f(x) = \frac{8}{\pi^2} \left\{ 2 \cdot \sin x + \frac{\pi}{4} \sin 2x - \frac{2}{9} \sin 3x - \frac{\pi}{16} \sin 4x + \frac{2}{25} \sin 5x \cdots \right\}$$

11.12

[풀이]

$$\therefore f(x) = (e^{\pi} - e^{-\pi}) \left[\frac{1}{2\pi} + \sum_{n=1}^{\infty} \left(\frac{(-1)^n}{\pi(n^2 + 1)} (\cos nx - n \sin nx) \right) \right]$$

11.13

[풀이]

$$\therefore f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \left(\frac{n}{1 - n^2} \cos \frac{n\pi}{2} \right) \sin nx$$

11.14

[풀이]

$$\therefore f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4(-1)^n}{n^2} \cos nx \right)$$

11.15

[풀이]

$$\therefore f(x) = 1 - \left\{ \frac{4}{\pi} \sin \pi x + \frac{4}{3\pi} \sin 3\pi x + \frac{4}{5\pi} \sin 5\pi x + \cdots \right\}$$

11.16

[풀이]

$$\therefore f(x) = -\frac{4k}{\pi} \left(\sin \frac{\pi}{7} x + \frac{1}{3} \sin \frac{3\pi}{7} x + \cdots \right)$$

11.17

[풀이]

$$\therefore f(x)z = \frac{3}{5}a + \frac{2a}{\pi} \left(\sin \frac{3\pi}{5} \cos \frac{\pi}{5} x + \frac{1}{2} \sin \frac{6\pi}{5} \cos \frac{2\pi}{5} x + \frac{1}{3} \sin \frac{9\pi}{5} \cos \frac{3\pi}{5} x + \cdots \right)$$

11.18

[풀이]

$$\begin{aligned}\therefore f(x) &= 2 - \frac{4}{\pi^2} \left(\cos \frac{\pi x}{2} + \frac{1}{9} \cos \frac{3\pi x}{2} + \frac{1}{25} \cos \frac{5\pi x}{2} \dots \right) \\ &\quad + \frac{2}{\pi} \left(2 \sin \frac{\pi x}{2} + \frac{1}{2} \sin \pi x + \frac{2}{3} \sin \frac{3\pi x}{2} \dots \right)\end{aligned}$$

11.19

[풀이]

$$\therefore f(x) = -\frac{3}{4} + \frac{12}{\pi^2} \left\{ 1 + \frac{1}{2} + \frac{1}{9} + \frac{1}{25} + \frac{1}{18} + \frac{1}{49} + \dots \right\}$$

11.20

[풀이]

$\therefore$ 우함수

11.21

[풀이]

$\therefore$ 속하지 않음

11.22

[풀이]

$\therefore$ 우함수

11.23

[풀이]

$\therefore$ 기함수

11.24

[풀이]

속하지 않음

11.25

[풀이]

$\therefore$ 우함수

11.26

[풀이]

속하지 않음

11.27

[풀이]

속하지 않음

11.28

[풀이]

푸리에 코사인급수 전개

$$f(x) = 1$$

푸리에 사인급수 전개

$$f(x) = \frac{4}{\pi} \left(\sin \frac{\pi}{4} x + \frac{1}{3} \sin \frac{3\pi}{2} x + \frac{1}{5} \sin \frac{5\pi}{2} x + \dots \right)$$

11.29

[풀이]

푸리에 코사인급수 전개

$$f(x) = 1 + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (\cos n\pi - 1) \cos \frac{n\pi x}{4}$$

푸리에 사인급수 전개

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (1 - 3\cos n\pi) \sin \frac{n\pi x}{4}$$

11.30

[풀이]

푸리에 코사인급수 전개

$$\rightarrow f(x) = \frac{1}{3} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (\cos n\pi + 1)$$

푸리에 사인급수 전개

$$\rightarrow f(x) = \frac{1}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \{ (16 - 2n^2\pi^2) \cos n\pi + 2n^2\pi^2 - 16 \} \sin \frac{n\pi x}{2}$$

11.31

[풀이]

푸리에 코사인급수 전개

$$\therefore f(x) = \frac{2(\sqrt{e^3} - 1)}{3} + \sum_{n=1}^{\infty} \frac{12}{4n^2\pi^2 + 9} \{ \sqrt{e^3}(-1)^n - 1 \} \cos \frac{n\pi}{3} x$$

푸리에 사인급수 전개

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{8n\pi}{4n^2\pi^2 + 9} \{ 1 - \sqrt{e^3}(-1)^n \} \sin \frac{n\pi}{3} x$$

11.32

[풀이]

푸리에 코사인급수 전개

$$\therefore f(x) = 1 + \frac{8}{\pi^2} \left(\cos \frac{\pi}{2} x + \frac{1}{3^2} \cos \frac{3\pi}{2} x + \frac{1}{5^2} \cos \frac{5\pi}{2} x + \dots \right)$$

푸리에 사인급수 전개

$$\therefore f(x) = \frac{4}{\pi} \left(\sin \frac{\pi}{2} x + \frac{1}{2} \sin \pi x + \frac{1}{3} \sin \frac{3\pi}{2} x + \dots \right)$$

11.33

[풀이]

푸리에 코사인급수 전개

$$\rightarrow f(x) = \frac{3}{8} \pi + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\cos \frac{n\pi}{2} - 1 \right) \cos nx$$

푸리에 사인급수 전개

$$\rightarrow f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{2}{n\pi} \sin \frac{n\pi}{2} - \cos n\pi \right) \sin nx$$