

[Chapter 13] 연습문제 정답

13.1

[풀이]

$$\therefore u(x, y) = c_1(y) + c_2(y)x$$

13.2

[풀이]

$$\therefore u(x, y) = A(y)e^x + B(y)e^{-x}$$

13.3

[풀이]

$$\therefore u(x, y) = f(x)e^{-y} + g(y)$$

13.4

[풀이]

$$\therefore u(x, y) = c_1(x)\cos 2y + c_2(x)\sin 2y$$

13.5

[풀이]

$$\therefore u(x, y) = c(y)e^{-x^2}$$

13.6

[풀이]

$$\therefore u(x, y) = c_1(x) + c_2(x)e^{-3y}$$

13.7

[풀이]

증명생략

13.8

[풀이]

(a) 증명생략

(b) $\therefore u = 2\sin(y - 3x)$

13.9

[풀이]

$$(a) \quad \therefore u = \frac{1}{6}x^3y^2 + h(y) + g(x)$$

$$(b) \quad \therefore u = \frac{1}{6}x^3y^2 + \cos y - \frac{1}{6}y^2 + x^2 - 1$$

13.10

[풀이]

$$\therefore u(x, t) = \frac{1}{2} \cos 3\pi t \sin 3\pi x$$

13.11

[풀이]

$$\therefore u(x, t) = \frac{4}{\pi^2} \left(\sin \pi x \cos \pi t + \frac{1}{27} \sin 3\pi x \cos 3\pi t + \frac{1}{125} \sin 5\pi x \cos 5\pi t + \dots \right)$$

13.12

[풀이]

$$\therefore u(x, t) = \frac{8}{\pi^2} \left(\frac{1}{4} \sin 2\pi x \cos 2\pi t - \frac{1}{36} \sin 6\pi x \cos 6\pi t + \frac{1}{100} \sin 10\pi x \cos 10\pi t + \dots \right)$$

13.13

[풀이]

$$\therefore u(x, t) = \frac{4}{\pi^3} \left\{ (4 - \pi) \cos \pi t \sin \pi x + \cos 2\pi t \sin 2\pi x + \frac{4 + 3\pi}{27} \cos 3\pi t \sin 3\pi x + \dots \right\}$$

13.14

[풀이]

$$\therefore u(x, y) = 4e^{-2(3x+y)}$$

13.15

[풀이]

$$\therefore u(x, t) = 100 \sin \frac{\pi x}{100} e^{-\alpha t}$$

13.16

[풀이]

$$\therefore G'' + \lambda^2 G = 0$$

$$H'' + k^2 H = 0$$

$$Q'' + q^2 Q = 0$$