

CHAPTER 01 연습문제 정답

1.1

$$(a) A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$$(b) x + y + z = 1$$

1.2

$$A + B = \begin{bmatrix} 5 & 5 \\ 2 & 4 \\ 4 & 13 \end{bmatrix}, \quad -2A = \begin{bmatrix} -8 & -10 \\ -4 & -6 \\ -2 & -18 \end{bmatrix}, \quad 3A - B = \begin{bmatrix} 11 & 15 \\ 6 & 8 \\ 0 & 23 \end{bmatrix}$$

1.3

$$(a) AB = \begin{bmatrix} 3 & 9 & -1 \\ -2 & 2 & -2 \end{bmatrix}, \quad BC = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$$

$$(b) AB = \begin{bmatrix} -8 & 7 \\ 37 & 8 \end{bmatrix}, \quad BA = \begin{bmatrix} -9 & 22 \\ 19 & 9 \end{bmatrix}$$

1.4

생략

1.5

생략

1.6

$$(a) x_1 = \frac{-22}{7}, \quad x_2 = \frac{39}{7}$$

$$(b) x_1 = 0, \quad x_2 = -3, \quad x_3 = -5$$

1.7

(a) 해가 무수히 많다.

(b) 해가 없다.

1.8

(a) $x_1 = \frac{155}{7}, x_2 = \frac{27}{7}, x_3 = -\frac{132}{35}$

(b) $x_1 = \frac{97}{53}, x_2 = \frac{942}{53}, x_3 = -\frac{61}{53}, x_4 = -\frac{272}{53}$

1.9

사자 : 5마리, 학 : 2마리

1.10

(a) $\begin{bmatrix} -5 & 4 & -3 \\ 5 & -16 & -18 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \\ 6 & 12 & 18 \end{bmatrix}$

1.11

(a) 거짓

(b) 거짓

CHAPTER 02 연습문제 정답

2.1

A : 삼각행렬, B : 대칭행렬, C : 대각행렬, D : 반대칭행렬

2.2

생략

2.3

$x = y = 1$ 또는 $x = y = 0$

2.4

생략

2.5

(a) $A^{-1} = \frac{1}{16} \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$

(b) $B^{-1} = \frac{1}{33} \begin{bmatrix} -6 & 5 \\ 9 & -2 \end{bmatrix}$

2.6

생략

2.7

생략

2.8

(a) 생략

(b) A 의 행렬식은 -81 이다.

2.9

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

2.10

$$(a) \ X = \frac{1}{2} \begin{bmatrix} 26 & -16 \\ 23 & 16 \end{bmatrix}$$

$$(b) \ X = \begin{bmatrix} 0 & -6 \\ 3 & 9 \end{bmatrix}$$

$$(c) \ X = -\frac{1}{25} \begin{bmatrix} -19 & 0 \\ 6 & 0 \\ -35 & -75 \end{bmatrix}$$

2.11

$$(a) \ x_1 = -\frac{13}{4}, \ x_2 = \frac{31}{4}$$

$$(b) \ x_1 = 0, \ x_2 = -3, \ x_3 = -5$$

2.12

$$(a) \ x_1 = -\frac{1}{35}, \ x_2 = \frac{135}{35}, \ x_3 = -\frac{132}{35}$$

$$(b) \ x_1 = 1, \ x_2 = 2, \ x_3 = 3, \ x_4 = 4$$

2.13

$$(a) \ tr(AB) = 4$$

$$(b) \ tr(BA) = 4$$

2.14

생략

2.15

생략

2.16

생략

CHAPTER 03 연습문제 정답

3.1

생략

3.2

생략

3.3

생략

3.4

(a) 선형독립

(b) 선형종속

3.5

생략

3.6

H 의 기저는 $\{\boldsymbol{x}_1, \boldsymbol{x}_2\}$ 이다.

3.7

$$\frac{1}{9}(2, -2, 1)$$

3.8

$$(a) \theta = \cos^{-1}\left(\frac{1}{14}\right)$$

$$(b) \theta = \cos^{-1}\left(\frac{6}{\sqrt{78}}\right)$$

3.9

$$x = 1$$

3.10

$$2 - 2\sqrt{2}$$

3.11

생략

3.12

$$\nabla f = \begin{bmatrix} 2x + 7y \\ 8y + 7x \end{bmatrix}, \quad \nabla f_{(2, 5)} = \begin{bmatrix} 39 \\ 51 \end{bmatrix}$$

3.13

$$\nabla f = \begin{bmatrix} 2x + y^2 \\ 2xy + z \\ y + 1 \end{bmatrix}, \quad \nabla f_{(1, 2, 3)} = \begin{bmatrix} 6 \\ 7 \\ 3 \end{bmatrix}$$

$$H(f) = \begin{bmatrix} 2 & 2y & 0 \\ 2y & 2x & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad H(f)_{(1, 2, 3)} = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

CHAPTER 04 연습문제 정답

4.1

- (a) L 은 선형변환이다.
- (b) L 은 선형변환이 아니다.

4.2

- (a) $\begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 1 & -2 \end{bmatrix}$
- (b) $\begin{bmatrix} 2 & 2 & -1 & 5 \\ 4 & 1 & 2 & 5 \end{bmatrix}$

4.3

L 은 일대일 변환이 아니고 위로의 변환이다.

4.4

생략

4.5

$P(2, 3\sqrt{2})$

4.6

$$\begin{bmatrix} 9 \\ 7 \\ 22 \end{bmatrix}$$

4.7

- (a) 랭크 : 2
- (b) 랭크 : 3

4.8

- (a) 랭크 : 3, 퇴화차수 : 1
- (b) 랭크 : 2, 퇴화차수 : 1

4.9

차원 = 3

4.10

랭크 : 3, 영공간 = $\text{span}\left\{\begin{bmatrix} 4 \\ -2 \\ -1 \\ 1 \end{bmatrix}\right\}$

4.11

- (a) 직교연산자이다.
- (b) 직교연산자이다.
- (c) 직교연산자이다.

4.12

생략

CHAPTER 05 연습문제 정답

5.1

(a) $\lambda = -2, 10$

(b) $\lambda = 3, 4$

5.2

(a) $(\lambda - 3)(\lambda + 1)$

(b) $(\lambda - 1)(\lambda - 2)(\lambda - 8)$

5.3

i) $\lambda_1 = 7$; $E_1 = \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$

ii) $\lambda_2 = \frac{-\sqrt{109}+5}{2}$; $E_2 = \text{span}\left\{\begin{bmatrix} \frac{-\sqrt{109}-3}{10} \\ 1 \\ 0 \end{bmatrix}\right\}$

iii) $\lambda_3 = \frac{\sqrt{109}+5}{2}$; $E_3 = \text{span}\left\{\begin{bmatrix} \frac{\sqrt{109}-3}{10} \\ 1 \\ 0 \end{bmatrix}\right\}$

5.4

특성다항식 : $(\lambda + 2)(\lambda - 3)(\lambda - 5)$

i) $\lambda_1 = -2$; $E_1 = \text{span}\left\{\begin{bmatrix} -35 \\ 12 \\ 19 \end{bmatrix}\right\}$

ii) $\lambda_2 = 3$; $E_2 = \text{span}\left\{\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}\right\}$

iii) $\lambda_3 = 5$; $E_3 = \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$

5.5

(a) i) $\lambda_1 = 2\sqrt{3} - 1$; $E_1 = \text{span}\left\{\begin{bmatrix} \sqrt{3}-1 \\ 1 \end{bmatrix}\right\}$

ii) $\lambda_2 = -2\sqrt{3} - 1$; $E_2 = \text{span}\left\{\begin{bmatrix} -\sqrt{3}-1 \\ 1 \end{bmatrix}\right\}$

(b) i) $\lambda_1 = -1$; $E_1 = \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$

ii) $\lambda_2 = 2$; $E_2 = \text{span}\left\{\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}\right\}$

5.6

(a) -18

(b) 5

(c) 7

5.7

[파이썬 코드를 이용한 실행 결과]

$A^{20} =$

```
[[ 24157816 -87403800  63245985]
 [ 63245985 -228826124 165580140]
 [165580140 -599074575 433494436]]
```

5.8

$$\lambda = k, (3 + \sqrt{5})k, (3 - \sqrt{5})k$$

5.9

(a) $\lambda^2 - 5\lambda + 11$

(b) $\lambda^3 - \lambda^2 - 15\lambda + 15$

5.10

(a) 특성다항식 : $(\lambda - 1)(\lambda - 2)^2$

i) $\lambda_1 = 1$, 고유벡터 = $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

ii) $\lambda_2 = 2$, 고유벡터 = $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $A^{10} = \begin{bmatrix} 1 & 0 & 1003 \\ 0 & 1004 & 0 \\ 0 & 0 & 1004 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

5.11

$tr(A) = 1, \det(A) = 18$

5.12

생략

CHAPTER 06 연습문제 정답

6.1

$$(a) \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 0 & 5 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & -1 & -1 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & -1 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$(c) \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \\ -2 & 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & -4 & 1 \end{bmatrix}$$

6.2

생략

6.3

$$\begin{bmatrix} \frac{1}{2} \\ 2 \\ 0 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ 2 \\ 1 \\ 4 \end{bmatrix} t \quad (t \text{는 임의의 상수})$$

6.4

$$(a) \quad x_1 = -\frac{6}{5}, \quad x_2 = \frac{1}{5}, \quad x_3 = \frac{3}{5}$$

$$(b) \quad x_1 = -1, \quad x_2 = -1, \quad x_3 = -1$$

6.5

$$x_1 = \frac{5}{6}, \quad x_2 = -\frac{1}{3}, \quad x_3 = -\frac{1}{2}$$

6.6

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

6.7

(a)

$$A = \begin{bmatrix} \frac{7\sqrt{5}}{20} & \frac{4\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{20} & \frac{4\sqrt{5}}{5} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]

A =

[[4 6]

[0 4]]

U=

[[0.89442719 -0.4472136]

[0.4472136 0.89442719]]

Sigma =

[[8. 0.]

[0. 2.]]

V^T =

[[0.4472136 0.89442719]

[-0.89442719 0.4472136]]

(b)

$$A = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]

A =

[[2 2]

[2 2]]

U=

[[-0.70710678 -0.70710678]

[-0.70710678 0.70710678]]

Sigma =

[[4. 0.]

[0. 0.]]

$V^T =$

```
[[ -0.70710678 -0.70710678]
 [ -0.70710678  0.70710678]]
```

(c)

[파이썬 코드를 이용한 실행 결과]

A =

```
[[1 1]
 [0 1]
 [1 0]]
```

U=

```
[[ -8.16496581e-01 -1.85577521e-16 -5.77350269e-01]
 [ -4.08248290e-01 -7.07106781e-01  5.77350269e-01]
 [ -4.08248290e-01  7.07106781e-01  5.77350269e-01]]
```

Sigma =

```
[[1.73205081 0.          ]
 [0.          1.          ]
 [0.          0.          ]]
```

$V^T =$

```
[[ -0.70710678 -0.70710678]
 [  0.70710678 -0.70710678]]
```

(d)

[파이썬 코드를 이용한 실행 결과]

A =

```
[[ -3 -1  2]
 [  2  1 -2]]
```

U=

```
[[ -0.7815437  0.6238505]
 [  0.6238505  0.7815437]]
```

Sigma =

```
[[4.77289369 0.          0.          ]
 [0.          0.46849315 0.          ]]
```

V^T =

```
[[ 7.52652862e-01  2.94453279e-01 -5.88906559e-01]
 [-6.58417549e-01  3.36596593e-01 -6.73193185e-01]
 [ 1.50028633e-16  8.94427191e-01  4.47213595e-01]]
```

(e)

$$A = \begin{bmatrix} 0 & \frac{\sqrt{5}}{2} & 0 \\ -\frac{2}{5} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2\sqrt{5} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]

A =

```
[[ 0  5  0]
 [-2  0  4]
 [ 0  0  0]]
```

U=

```
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
```

Sigma =

```
[[5.      0.      0.      ]
 [0.      4.47213595 0.      ]
 [0.      0.      0.      ]]
```

V^T =

```
[[ -0.      1.      0.      ]
 [-0.4472136  0.      0.89442719]
 [-0.89442719  0.     -0.4472136 ]]
```

6.8

(a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$(c) \ A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

6.9

$$(a) \ A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad A A^T = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

i) $A^T A$ 의 특잇값 분해 결과

[파이썬 코드를 이용한 실행 결과]

A =

[[2 1]

[1 1]]

U=

[[-0.85065081 -0.52573111]

[-0.52573111 0.85065081]]

sigma=

[[2.61803399 0.]

[0. 0.38196601]]

V^T =

[[-0.85065081 -0.52573111]

[-0.52573111 0.85065081]]

ii) $A A^T$ 의 특잇값 분해 결과

$$A A^T = \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}^T$$

$$(b) \ A^T A = \begin{bmatrix} 13 & 0 & 6 \\ 0 & 0 & 0 \\ 6 & 0 & 4 \end{bmatrix}, \ AA^T = \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix}$$

i) $A^T A$ 의 특잇값 분해 결과

$$A^T A = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix} \begin{bmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix}^T$$

ii) AA^T 의 특잇값 분해 결과

$$AA^T = \begin{bmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix} \begin{bmatrix} 13 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix}^T$$

6.10

[파이썬 코드를 이용한 실행 결과]

A =

```
[[ -3  -1   2]
 [  2   1  -2]]
```

Pseudo inverse of A =

```
[[ -1.  -1. ]
 [  0.4  0.6]
 [-0.8 -1.2]]
```

CHAPTER 07 연습문제 정답

7.1

- (a) $f'(x) = 77x^{76} - 2x + 1$
- (b) $f'(x) = 4x^3 - 21x^2$
- (c) $f'(x) = 5x^4 + 2e^{2x}$
- (d) $f'(x) = \sqrt{3}$
- (e) $f'(x) = -\frac{1}{2x\sqrt{x}} + \frac{1}{5\sqrt[5]{x^4}}$

7.2

- (a) $f'(x) = 3\cos x + 5\sin x$
- (b) $f'(x) = 2x\sin x + x^2\cos x$
- (c) $f'(x) = \frac{\cos x}{(1 + \cos x)^2}$
- (d) $f'(x) = \frac{1 - x\sec x}{\sec x + \tan x}$
- (e) $f'(x) = e^x(\cos x - \sin x)$

7.3

- (a) $y = 4x + 4$
- (b) $y = 4x + 3$
- (c) $y = \frac{5}{2}x - \frac{3}{2}$
- (d) $y = x + 1$
- (e) $y = \pi$

7.4

- (a) 증명 생략, $c = 1$
- (b) 증명 생략, $c = \frac{\pi}{2}$
- (c) 증명 생략, $c = 1$

7.5

- (a) 증명 생략, $c = \frac{3}{2}$
- (b) 증명 생략, $c = 0, \pi, 2\pi$
- (c) 증명 생략, $c = 4$

7.6

- (a) 증명 생략, 극한값 $= \frac{1}{6}$
- (b) 증명 생략, 극한값 $= 2$
- (c) 증명 생략, 로피탈 정리 적용 불가능
- (d) 증명 생략, 극한값 $= 0$
- (e) 증명 생략, 극한값 $= \frac{\log 7 - \log 5}{\log 4 - \log 3}$

7.7

생략

7.8

- (a) $f'(x) = 10(x^7 + 3x^3 - 5x)^9(7x^6 + 9x^2 - 5)$
- (b) $f'(x) = 10(5x + 9)(x^2 - x + 7)^9(11x^2 + 12x - 2)$
- (c) $f'(x) = -\sin x \cos(\cos x)$
- (d) $f'(x) = (\sin 2x + 2x \cos 2x)e^{x \sin 2x} + 3x^2 2^{x^3} \log 2$
- (e) $f'(x) = 2x(x + 1)e^{2x} + \frac{7^{\sqrt{x}} \log 7}{2\sqrt{x}}$

7.9

생략

7.10

$$a = -7, b = -12$$

CHAPTER 08 연습문제 정답

8.1

생략

8.2

$$-2\cos x + C$$

8.3

$$\frac{1}{545}(x^5 + 5x + 1)^{109} + C$$

8.4

$$\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3}$$

8.5

$$2e^{\sqrt{x}} + C$$

8.6

$$(a) \frac{1}{4}x^2(2\log x - 1)$$

$$(b) \frac{1}{2}e^x(\sin x + \cos x)$$

8.7

$$(a) 54$$

$$(b) \frac{10}{3}$$

$$(c) \log \frac{19}{7}$$

$$(d) 0$$

8.8

생략

8.9

(a) 102

(b) 1

8.10

(a) $\frac{3\pi}{2}$

(b) $\frac{1}{54}(109\sqrt{109} - 37\sqrt{37})$

8.11

(a) π

(b) π

8.12

(a) $\frac{4}{3}\sqrt{2}(5\sqrt{5} - 1)\pi$

(b) $\frac{17\sqrt{17} - 27}{6}\pi$

CHAPTER 09 연습문제 정답

9.1

(a) 정의역 : $\mathbb{R}^2$, 치역 : $\mathbb{R}$

(b) 정의역 : $D = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 < 1\}$, 치역 : $(-\infty, 0]$

9.2

(a) $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = k\}$

(b) $\{(x, y) \in \mathbb{R}^2 \mid y - x^2 = k\}$

9.3

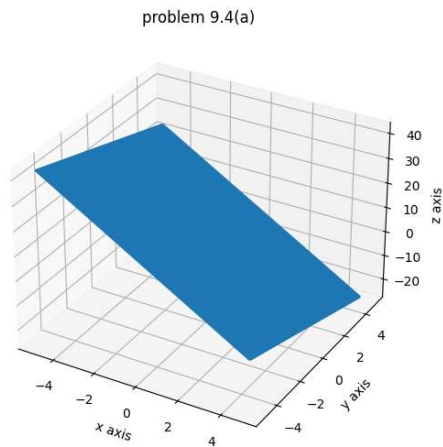
$$\frac{\partial z}{\partial u} = 3((v + e^u)^2 + u^2 v)^2 (2uv + 2e^u(v + e^u)) + 4u^3 v^2 (v + e^u) + u^4 v^2 e^u$$

$$\frac{\partial z}{\partial v} = 3((v + e^u)^2 + u^2 v)^2 (2v + 2e^u + u^2) + 2u^4 v (v + e^u) + u^4 v^2$$

9.4

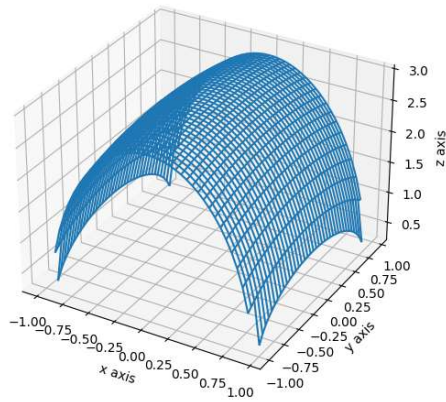
[파이썬 코드를 이용한 실행 결과]

(a)



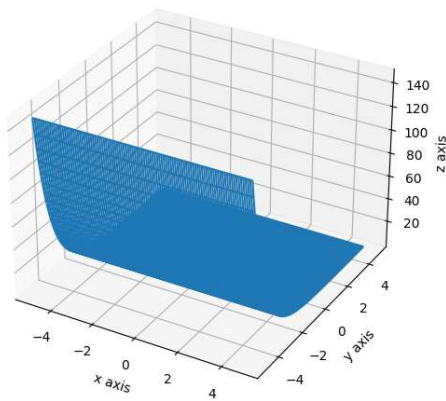
(b)

problem 9.4(b)



(c)

problem 9.4(c)



9.5

(a) 2

(b) 0

(c) $-\frac{11}{633}$

9.6

(a) $\mathbb{R}^2$

(b) $\{(x, y) \in \mathbb{R}^2 \mid y \neq 0\}$

(c) $\{(x, y) \in \mathbb{R}^2 \mid x + 3y \geq 0\}$

(d) $\{(x, y) \in \mathbb{R}^2 \mid x \neq y\}$

9.7

- (a) $\frac{\partial f}{\partial x} = -4x, \quad \frac{\partial f}{\partial y} = 6y^5 - 4y$
- (b) $\frac{\partial f}{\partial x} = \sec^2(x+y), \quad \frac{\partial f}{\partial y} = \sec^2(x+y)$
- (c) $\frac{\partial f}{\partial x} = \frac{e^y}{(x+y^2)^2}, \quad \frac{\partial f}{\partial y} = \frac{e^y(x+(y-2)y)}{(x+y^2)^2}$
- (d) $\frac{\partial f}{\partial x} = \frac{x}{y\sqrt{x^2+y^2}+x^2+y^2}, \quad \frac{\partial f}{\partial y} = \frac{1}{\sqrt{x^2+y^2}}$

9.8

- (a) $(3x^2y^6 + 5x^4y^4, 6x^3y^5 + 4x^5y^3)$
- (b) $(3\cos(3x+7y), 7\cos(3x+7y))$
- (c) $(e^{-y}, -xe^{-y})$
- (d) $\left(\frac{1}{x+3y}, \frac{3}{x+3y}\right)$

9.9

- (a) $\begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$
- (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- (c) $\begin{bmatrix} 2 & -2 & 4 \\ -2 & 2 & 4 \\ 4 & 4 & 4 \end{bmatrix}$

9.10

- (a) 부정부호행렬
- (b) 음의 준정부호행렬
- (c) 양의 준정부호행렬

CHAPTER 10 연습문제 정답

10.1

(a) $\frac{23}{40}$

(b) $\frac{1}{4}$

(c) $\frac{5}{8}$

10.2

(a) 독립이 아니다.

(b) 독립이다.

(c) 독립이 아니다.

(d) 독립이 아니다.

(e) 독립이 아니다.

10.3

$$p = \frac{1}{6}$$

10.4

(a) 0.525

(b) 0.175

10.5

0.016

10.6

0.036

10.7

$$(a) f_X(x) = \begin{cases} \frac{4^x e^{-4}}{x!}, & x = 0, 1, 2, \dots \\ 0 & , \text{그 외} \end{cases}$$

$$(b) 0.762$$

$$(c) E(X) = 4, V(X) = 4$$

10.8

680명

10.9

$$\mu_Y = a\mu_X + b, \sigma_Y^2 = (a\sigma_X)^2, \sigma_Y = a\sigma_X$$

10.10

생략

CHAPTER 11 연습문제 정답

11.1

생략

11.2

생략

11.3

(a) 4

(b) $E(Y) = -5$, $V(Y) = 36$

11.4

생략

11.5

생략

11.6

(a) $E(X) = \frac{2}{3}$, $V(X) = \frac{2}{9}$

(b) $E(Y) = \frac{3}{4}$, $V(Y) = \frac{11}{16}$

(c) $E(XY) = \frac{13}{24}$

(d) $\text{Cov}(X, Y) = \frac{1}{24}$

(e) $V(X+Y) = \frac{143}{144}$

11.7

(a) $k = 27$

(b)

Y X	0	1	2	$P(Y = y_j)$
0	0	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{9}$
1	$\frac{2}{27}$	$\frac{3}{27}$	$\frac{4}{27}$	$\frac{1}{3}$
2	$\frac{4}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{5}{9}$
$P(X = x_i)$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{4}{9}$	1

(c) $\text{Corr}(X, Y) = 0.184$

11.8

$y = 1.29 + 1.40x$

11.9

$y = 1.83 + 0.86x + 0.17y$

11.10

$y = 5.20 + 0.40x, \quad y(9) = 8.80$

11.11

$y = -0.76 + 0.11x, \quad y(30) = 2.54$

CHAPTER 12 연습문제 정답

12.1

$$\frac{2}{3}$$

12.2

혼동행렬은 다음과 같다.

		예측된 클래스	
		양성	음성
실제 클래스	양성	11	4
	음성	4	0

$$(\text{정밀도}) = \frac{11}{15}, \quad (\text{재현율}) = \frac{11}{15}, \quad (\text{F1 점수}) = \frac{11}{30}$$

12.3

$$(\text{평균절대오차}) = 1, \quad (\text{평균제곱오차}) = 1.33$$

$$(\text{평균제곱근오차}) = 1.15, \quad (R^2 \text{ 점수}) = 0.80$$

12.4

[파이썬 코드를 이용한 실행 결과]

입력 데이터의 라벨 = ['배']

12.5

i) 엔트로피를 이용한 정보 이득 함수의 값 = 0.24

ii) 지니 불순도를 이용한 정보 이득 함수의 값 = 0.13

12.6

[파이썬 코드를 이용한 실행 결과]

투영행렬 W =

[[0.59140322]

[-0.57633338]

[-0.56398766]]

12.7

$$y = 1.25 + 1.37x + 0.01x^2$$

12.8

$$\mu_1 = (-1, 0), \mu_2 = (1.75, 1.75)$$

CHAPTER 13 연습문제 정답

13.1

$$(a) \tau(\mathbf{w}\mathbf{x}_1^T) = -1, \sigma(\mathbf{w}\mathbf{x}_1^T) \doteq 0.01, \text{ReLU}(\mathbf{w}\mathbf{x}_1^T) = 0$$

$$(b) \tau(\mathbf{w}\mathbf{x}_2^T) = 1, \sigma(\mathbf{w}\mathbf{x}_2^T) \doteq 1.00, \text{ReLU}(\mathbf{w}\mathbf{x}_2^T) = 13$$

13.2

$$\sigma(\mathbf{w}\mathbf{x}_0^T) = \sigma(2) = \frac{1}{1 + e^{-2}} \doteq 0.88$$

$$\text{ReLU}(\mathbf{w}\mathbf{x}_0^T) = \text{ReLU}(2) = 2$$

13.3

$$(a) \tau(\mathbf{w}\mathbf{x}_1^T) = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \sigma(\mathbf{w}\mathbf{x}_1^T) \doteq (0.01, 1.00, 0.01), \text{ReLU}(\mathbf{w}\mathbf{x}_1^T) = (0, 5, 0)$$

$$(b) \tau(\mathbf{w}\mathbf{x}_2^T) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \sigma(\mathbf{w}\mathbf{x}_2^T) \doteq (1.00, 0.88, 1.00), \text{ReLU}(\mathbf{w}\mathbf{x}_2^T) = (13, 2, 30)$$

13.4

$$(0.00, 0.01, 0.99)$$

13.5

$$\mathbf{c} = (\cdots, 0, -9, 22, 0, -8, 3, 31, 0, -5, 0, \cdots)$$

13.6

$$n_{out}^2 = 168$$

13.7

$$C = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

13.8

$$\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$