

Section 10.1 연습문제

1.

 $y = \tan(x + C)$

2.

 $3(x + C)y^3 = 1$

3.

 $\ln|2y - 3| = 2x + C$

4.

 $(y + 1)^2 = Ce^{2x - x^2}$

5.

 $x(1 - \sqrt{y})^2 = C$

6.

 $y^2 = x^2 + 4\ln|x| + C$

7.

 $x^2 + y^2 \ln|x| + Cy^2 = 0$

8.

 $xy^3 - x^3y = C$

9.

 $x = y \ln|x| + Cy$

10.

 $y = x \ln|x| + Cx$

11.

 $\frac{x^2y + x}{y} + \ln y = C$

12.

 $x^2 \sin y - y = C$

13.

 $2y^2 - \frac{1}{x^2 y^2} = C$

14.

 $x^2 - 2y^2 + 2x \sin y = C$

15.

 $y^2 \sin x = C$

16.

 $2 \sin x + \sin y = C$

17.

 $\ln |x| + y = 1$

18.

 $y = \frac{3}{2}x$

19.

 $y = \frac{e^x}{-2 + e^x}$

20.

 $\tan^{-1} y = \ln |x| + \frac{\pi}{4} - 1$

21.

 $y^4 (2x^4 + y^4) = 1$

22.

 $y^4 = 2x^8 - x^4$

23.

 $\sqrt{xy} = x + 1 - \sqrt{2}$

24.

 $\sqrt{y^2 - x^2} = 2 - y$

25.

 $5x^3y + 2y^5 = 0$

26.

 $e^{xy} + 2x - y^2 = 1$

27.

 $x^2 + 3\ln y = 9$

28.

 $2x \sin x + 2\cos x + \sin y = 2$

29.

 $y^2(1 - x^2) + \sin^2 x = 4$

30.

 $\cosh x \cos y = 1$

31.

 $y = 1 + \frac{C}{\sqrt{x^2 - 4}}$

32.

 $y = x + C\sqrt{x^2 + 1}$

33.

 $xy = 2x \sin x - (x^2 - 2) \cos x + C$

34.

 $xy = \sin x - x \cos x + C$

35.

 $y(x-2) = x^3 - 3x^2 + 4$

36.

 $y = \cos x(-2\cos x + 3)$

37.



(a) $i(t) = \frac{5}{2}(1 - e^{-4t})$

(b) $\lim_{t \rightarrow \infty} i(t) = \frac{5}{2}$

38.



(a) $q(t) = 6(1 - e^{-2t})$

(b) $i(t) = 12e^{-2t}$

Section 10.2 연습문제

1.

 $y = C_1 e^x + C_2 e^{2x}$

2.

 $y = C_1 e^{-5x} + C_2 e^{6x}$

3.

 $y = C_1 e^{-3x} + C_2 e^{-7x}$

4.

 $y = C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x$

5.

 $y = (C_1 + C_2 x) e^x$

6.

 $y = e^{-2x} (C_1 \cos \pi x + C_2 \sin \pi x)$

7.

 $y = e^{-3x} (C_1 + C_2 x)$

8.

 $y = C_1 e^{-(1+\sqrt{3})x} + C_2 e^{(-1+\sqrt{3})x}$

9.

 $y = e^{-x} (C_1 \cos x + C_2 \sin x)$

10.

 $y = e^{3x/2} \left(C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right)$

11.

 $y = e^{-x} (C_1 \cos \sqrt{2} x + C_2 \sin \sqrt{2} x)$

12.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = e^x (C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x)$$

13.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = e^{2x} (C_1 \cos 3x + C_2 \sin 3x)$$

14.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = C_1 e^{-(1+\sqrt{2})x} + C_2 e^{(-1+\sqrt{2})x}$$

15.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = C_1 e^{2x} + C_2 e^{-x} - x + 1$$

16.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = C_1 e^{(-2+\sqrt{6})x} + C_2 e^{-(2+\sqrt{6})x} - \frac{1}{2}x^2 - x - \frac{9}{2}$$

17.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = C_1 e^x + C_2 e^{2x} + \frac{5}{2}e^{3x}$$

18.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = e^{3x/2} \left(C_1 \cos \frac{\sqrt{7}}{2}x + C_2 \sin \frac{\sqrt{7}}{2}x \right) + \frac{1}{7}e^{-2x}$$

19.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = e^{-x} (C_1 \cos \sqrt{3} x + C_2 \sin \sqrt{3} x) + \frac{1}{32}(2x-1)e^{3x}$$

20.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = (C_1 + C_2 x + x^2)e^{-x} + \frac{1}{4}e^x$$

21.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = (C_1 + C_2 x)e^{-x} + \frac{1}{4}e^x + \frac{1}{2}x^2 e^{-x}$$

22.

$$\textcircled{\text{II}} \textcircled{\text{O}} y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{4} \sin x$$

23.

$$\textcircled{\text{풀이}} \quad y = e^{x/2} \left(C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right) + \frac{6}{73} \cos 3x - \frac{16}{73} \sin 3x$$

24.

$$\textcircled{\text{풀이}} \quad y_h = C_1 e^{-x} + C_2 e^{3x} - \frac{1}{13} (2 \cos 2x + 29 \sin 2x)$$

25.

$$\textcircled{\text{풀이}} \quad y_h = C_1 e^x + C_2 e^{3x} + \frac{1}{50} [(5x-2) \cos x - (10x+11) \sin x]$$

26.

$$\textcircled{\text{풀이}} \quad y_h = e^{-x/2} \left(C_1 \cos \frac{\sqrt{7}}{2} x + C_2 \sin \frac{\sqrt{7}}{2} x \right) + (-x+3) \cos x + (x-3) \sin x$$

27.

$$\textcircled{\text{풀이}} \quad y = e^{-x} (C_1 \cos x + C_2 \sin x) + \frac{1}{4} e^x (\cos x + \sin x)$$

28.

$$\textcircled{\text{풀이}} \quad y = e^x (C_1 \cos x + C_2 \sin x) - \frac{1}{3} e^x \sin 2x$$

29.

$\textcircled{\text{풀이}}$

(a) e^x 을 테일러 급수전개식으로 나타내면 $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ 이므로 x 를 iqx 로 대치하면 된다.

(b) $i^2 = -1$, $i^3 = -i$, $i^4 = 1$ 이므로

$$\begin{aligned} e^{iqx} &= \sum_{n=0}^{\infty} \frac{(iqx)^{4n}}{n!} + \sum_{n=0}^{\infty} \frac{(iqx)^{4n+1}}{n!} + \sum_{n=0}^{\infty} \frac{(iqx)^{4n+2}}{n!} + \sum_{n=0}^{\infty} \frac{(iqx)^{4n+3}}{n!} \\ &= \sum_{n=0}^{\infty} \frac{(qx)^{4n}}{n!} + i \sum_{n=0}^{\infty} \frac{(qx)^{4n+1}}{n!} - \sum_{n=0}^{\infty} \frac{(qx)^{4n+2}}{n!} - i \sum_{n=0}^{\infty} \frac{(qx)^{4n+3}}{n!} \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n (qx)^{2n}}{(2n)!} + i \sum_{n=0}^{\infty} \frac{(-1)^n (qx)^{2n+1}}{(2n+1)!} \end{aligned}$$

(c) $\cos qx$ 와 $\sin qx$ 의 테일러 급수전개식을 구하면 각각

$$\cos qx = \sum_{n=0}^{\infty} \frac{(-1)^n (qx)^{2n}}{(2n)!}, \quad \sin qx = \sum_{n=0}^{\infty} \frac{(-1)^n (qx)^{2n+1}}{(2n+1)!}$$

이 고, $e^{(p+iq)x} = e^{px} \cdot e^{iqx}$ 이므로 명백히 $e^{(p+iq)x} = e^{px}(\cos qx + i \sin qx)$ 가 성립한다.

Section 10.3 연습문제

1.

 풀이

(a) $P(t) = M(1 - e^{-kt})$

(b) $\lim_{t \rightarrow \infty} P(t) = M$

2.

 풀이

(a) (i) $a \neq b$ 인 경우 : $c(t) = \frac{aCe^{(b-a)kt} - b}{Ce^{(b-a)kt} - 1}$

(ii) $a = b$ 인 경우 : $c(t) = a - \frac{1}{kt + C}$

(b) (i) $a \neq b$ 인 경우 : $c(t) = \frac{ab(e^{(b-a)kt} - 1)}{be^{(b-a)kt} - a}$

(ii) $a = b$ 인 경우 : $c(t) = \frac{a^2kt}{akt + 1}$

3.

 풀이

(a) $y = 125 + 75 \exp^{-t/100}$

(b) 125lb가 된다.

4.

 풀이

$$t = \frac{1.3862}{0.9163} = 1.51$$

5.

 풀이

앞으로 약 28년 후에 40년 전의 인구보다 2배로 늘어나게 된다.

6.

 풀이

(a) $P(t) = \frac{aCe^{at}}{1 + bCe^{at}}$

(b) $P(t) = \frac{ap_0e^{at}}{a - bp_0 + bp_0e^{at}}$

$$(c) \lim_{t \rightarrow \infty} P(t) = \frac{a}{b}$$

7.

 풀이

$$(a) P(t) = e^{(a - Ce^{-bt})/b}$$

$$(b) P(t) = e^{(a - (a - b \ln P_0) e^{-bt})/b}$$

$$(c) \lim_{t \rightarrow \infty} P(t) = e^{a/b}$$

8.

 풀이

$$(a) i(t) = \frac{6}{61}(-6 \cos 60t + 5 \sin 60t) + \frac{97}{61} e^{-50t}$$

$$(b) i\left(\frac{\pi}{90}\right) = \frac{6}{61}\left(3 + \frac{5\sqrt{3}}{2}\right) + \frac{97}{61} e^{-5\pi/9} \approx 0.9986$$

9.

 풀이

$$(a) q(t) = -\frac{6}{90001}(300 \cos 60t - \sin 60t) + \frac{1800}{90001} e^{-t/5}$$

$$(b) i(t) = \frac{360}{90001}(\cos 60t + 300 \sin 60t) - \frac{360}{90001} e^{-t/5}$$

10.

 풀이

θ 에 대한 진자운동의 일반해는 다음과 같다.

$$\theta(t) = \sqrt{C_1^2 + C_2^2} \sin\left(\sqrt{\frac{g}{L}} t + \alpha\right), \quad \tan \alpha = \frac{C_2}{C_1}$$

따라서 이 운동의 주기는 $T = \frac{2\pi}{\sqrt{g/L}} = 2\pi \sqrt{\frac{L}{g}}$ 이다.

11.

 풀이

$$(a) \quad q(t) = \frac{1}{10}e^{-t}(\cos t - 3\sin t) - \frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$$

$$i(t) = \frac{1}{10}e^{-t}[-4\cos t + 2\sin t] + \frac{1}{5}\sin 2t + \frac{2}{5}\cos 2t$$

$$(b) \quad q(t) = e^{-t}\left(\frac{23}{1885}\cos 2t + \frac{173}{3770}\sin 2t\right) - \frac{23}{1885}\cos 10t - \frac{3}{377}\sin 10t$$

$$i(t) = e^{-t}\left(\frac{30}{377}\cos 2t - \frac{53}{754}\sin 2t\right) + \frac{46}{377}\sin 10t - \frac{30}{377}\cos 10t$$

12.

 풀이

$$q(t) = 1.156q_0 e^{-25t} \sin(43.3t + 1.045)$$

13.

 풀이

$$(a) \quad \text{이 운동은 진폭 } 1/6(\text{ft}), \text{ 주기 } 2\pi/\sqrt{96} = 0.64(\text{sec}), \text{ 진동수 } \frac{\sqrt{96}}{2\pi} = 1.56(\text{cycle/sec}) \text{인}$$

단순진동이다.

$$(b) \quad x(t) = e^{-0.0156t}\left(\frac{1}{6}\cos 9.8t + 0.000265\sin 9.8t\right)$$

(c) 시간이 경과됨에 따라 평형위치에 놓인다.

(d) 시간이 충분히 경과하면 감쇠진동은 소멸되고 단진동만 남게 된다.

Section 10.4 연습문제

1.

$$\textcircled{\text{풀이}} \quad L[2t^2 + t + 3] = \frac{3s^2 + s + 4}{s^3}, \quad s > 0$$

2.

$$\textcircled{\text{풀이}} \quad L[e^{t-2}] = \frac{e^{-2}}{s-1}, \quad s > 1$$

3.

$$\textcircled{\text{풀이}} \quad L[\sin 2t] = \frac{2}{s^2 + 4}, \quad s > 0$$

4.

$$\textcircled{\text{풀이}} \quad L[\sin t + \cos t] = \frac{s+1}{s^2+1}, \quad s > 0$$

5.

$$\textcircled{\text{풀이}} \quad L[e^{2t} \sin t] = \frac{1}{s^2 - 4s + 5}, \quad s > 2$$

6.

$$\textcircled{\text{풀이}} \quad L[f(t)] = \frac{s^2 + 1 - 2(s^2 + 1)e^{-\pi s} + (s^2 + s + 1)e^{-2\pi s}}{s(s^2 + 1)}, \quad s > 0$$

7.

$$\textcircled{\text{풀이}} \quad L[t^3 e^{2t/3}] = \frac{1}{s^2 - 4s + 5}, \quad s > \frac{2}{3}$$

8.

$$\textcircled{\text{풀이}} \quad L[(t-2)^3 u_2(t)] = \frac{6e^{-2s}}{s^4}, \quad s > 0$$

9.

$$\textcircled{\text{풀이}} \quad L^{-1}\left[\frac{1}{(s-2)(s-1)(s+1)(s+2)}\right] = \frac{1}{6}(\sinh 2t - 2\sinh t)$$

10.

$$\textcircled{\text{II}} L^{-1} \left[\frac{s+1}{s^2(s+2)} \right] = \frac{1}{4} (2t - 1 - e^{-2t})$$

11.

$$\textcircled{\text{II}} L^{-1} \left[\frac{s-1}{(s^2+1)(s^2+4)} \right] = \frac{1}{3} \left(\cos t - \sin t - \frac{1}{2} \sin 2t + \cos 2t \right)$$

12.

$$\textcircled{\text{II}} L^{-1} \left[\frac{(s-2)e^{-s}+1}{s(s-2)} \right] = u_1(t) + e^{2t}$$