

Section 8.1 연습문제

1.

 $\int_0^2 \int_{-1}^2 (x + y + 2) dx dy = 21$

2.

 $\int_0^1 \int_{x^2}^x (x^2 + y^3) dy dx = \frac{13}{180}$

3.

 $\int_0^{\pi/2} \int_{-\pi}^{\pi} (\sin x + \cos y) dx dy = 2\pi$

4.

 $\int_0^{\pi} \int_{-2y}^y y \sin x dx dy = 2$

5.

 $\int_0^{\ln 2} \int_0^{\ln y} e^{2x+y} dx dy = -\frac{1}{2} + (-1 + \ln 2)^2$

6.

 $\int_0^2 \int_1^x \frac{x e^x}{y^2} dy dx = 2$

7.

 $\int_0^2 \int_0^1 \frac{1}{(x+1)(y+1)} dy dx = (\ln 2)(\ln 3)$

8.

 $\int_1^2 \int_y^{y^2+1} \left(\frac{x}{y^2} + \frac{y}{x^2} \right) dx dy = \frac{1}{126} \left(35 - 6 \ln \frac{5}{2} \right)$

9.

 $\int_0^{\pi/2} \int_0^{\cos y} e^{\sin y} dx dy = e - 1$

10.

$$\textcircled{\text{II}} \int_0^1 \int_0^{y^2} 2ye^x dx dy = e - 2$$

11.

$$\textcircled{\text{II}} \int_0^1 \int_0^y \sqrt{1-x^2} dx dy = \frac{3\pi-4}{12}$$

12.

$$\textcircled{\text{II}} \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2} dy dx = \frac{16\pi}{3}$$

13.

$$\textcircled{\text{II}} \int_0^1 \int_{\sqrt{y}}^1 \frac{\sin x}{x} dx dy = \sin 1 - \cos 1$$

14.

$$\textcircled{\text{II}} \int_0^1 \int_x^1 \sin y^2 dy dx = \frac{1}{2}(1 - \cos 1)$$

15.

$$\textcircled{\text{II}} \int_0^4 \int_{x/2}^2 e^{y^2} dy dx = e^4 - 1$$

16.

$$\textcircled{\text{II}} \int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy = \frac{e^8 - 1}{4}$$

17.

$$\textcircled{\text{II}} \int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy dx = \frac{e^8 - 1}{4}$$

18.

$$\textcircled{\text{II}} \int_0^{\ln 9} \int_{e^x}^9 \frac{1}{\ln y} dy dx = 8$$

19.

$$\textcircled{\text{II}} \int_0^1 \int_0^{\cos^{-1}y} \sin x \sqrt{1+\sin^2 x} dx dy = \frac{-1+2\sqrt{2}}{3}$$

20.

 $\int_0^1 \int_{\sin^{-1} y}^{\pi/2} \cos x \sqrt{1 + \cos^2 x} \, dx \, dy = \frac{-1 + 2\sqrt{2}}{3}$

21.

 $A = 8$

22.

 $A = \frac{9}{2}$

23.

 $A = 9$

24.

 $A = 2 - 2\ln 2$

25.

 $\iint_D f(x, y) \, dV = 1 + \sqrt[3]{2}$

26.

 $A = \frac{8}{3}$

27.

 $A = \frac{8}{3}$

28.

 $A = 2$

29.

 $A = 2(\sqrt{2} - 1)$

Section 8.2 연습문제

1.

 $\int_{-2}^2 \int_0^3 \int_{-1}^2 2 \, dz \, dy \, dx = 72$

2.

 $\int_0^1 \int_0^2 \int_0^3 4 \, dy \, dx \, dz = 24$

3.

 $\int_0^\pi \int_0^\pi \int_0^\pi \sin(x+y+z) \, dx \, dy \, dz = -8$

4.

 $\int_0^\pi \int_0^\pi \int_0^\pi (x+y+z) \sin(x+y+z) \, dx \, dy \, dz = -12\pi$

5.

 $\int_1^5 \int_{-2x}^x \int_{2y}^{x-2} 2 \, dz \, dy \, dx = 352$

6.

 $\int_{-1}^1 \int_0^z \int_{z-1}^{2z+1} 2 \, dx \, dy \, dz = \frac{4}{3}$

7.

 $\int_{-1}^1 \int_{x^2}^x \int_0^{xy} 8xyz \, dz \, dy \, dx = 0$

8.

 $\int_0^1 \int_{x^2-1}^{2x} \int_{x-z}^{x+z} 2yz \, dy \, dz \, dx = \frac{23}{10}$

9.

 $\int_0^2 \int_0^1 \int_0^{\sqrt{1-z^2}} xze^y \, dx \, dz \, dy = \frac{e^2-1}{8}$

10.

$$\textcircled{\frac{\pi}{2}} \int_1^{3e} \int_1^{2e} \int_1^e \frac{1}{xyz} dz dy dx = \ln(2e) \ln(3e)$$

11.

$$\textcircled{\frac{\pi}{2}} \int_0^{\pi/3} \int_0^{\ln(\tan z)} \int_{-\infty}^y e^{2x} dx dy dz = \frac{\sqrt{3}}{4} - \frac{\pi}{6}$$

12.

$$\textcircled{\frac{\pi}{2}} \int_1^e \int_1^e \int_1^e \ln x \ln y \ln z dx dy dz = 1$$

13.

$$\textcircled{\frac{\pi}{2}} \int_0^\infty \int_0^\infty \frac{1}{(1+x)^2 (1+y)^2} dx dy = 1$$

14.

$$\textcircled{\frac{\pi}{2}} \int_0^\infty \int_0^\infty e^{-(x+y)} dx dy = 1$$

15.

$$\textcircled{\frac{\pi}{2}} \int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} dz dy dx = \frac{1}{12}$$

16.

$$\textcircled{\frac{\pi}{2}} \int_0^1 \int_0^{1-x^2} \int_0^{1-x} dz dy dx = \frac{5}{12}$$

17.

$$\textcircled{\frac{\pi}{2}} \int_0^1 \int_{-1}^1 \int_0^{y^2} dz dy dx = \frac{2}{3}$$

18.

$$\textcircled{\frac{\pi}{2}} 2 \int_0^1 \int_{-\sqrt{1-x^2}}^0 \int_0^{-y} dz dy dx = \frac{2}{3}$$

19.

$$\textcircled{\frac{\pi}{2}} a = 3$$

Section 8.3 연습문제

1.

 $\int_0^{\pi/2} \int_0^2 r^2 \sin \theta \, dr \, d\theta = \frac{8}{3}$

2.

 $\int_0^{2\pi} \int_0^1 r (\sin \theta + \cos \theta) \, dr \, d\theta = 0$

3.

 $\int_0^{\pi/2} \int_0^{1+\cos \theta} r^2 \sin \theta \, dr \, d\theta = \frac{5}{4}$

4.

 $\int_0^{\pi/2} \int_0^{\sin \theta} (\sin \theta + \cos \theta) \, dr \, d\theta = \frac{2+\pi}{4}$

5.

 $\int_0^{\pi/4} \int_{1-\cos \theta}^{1-\sin \theta} r^2 \sin \theta \cos \theta \, dr \, d\theta = \frac{11}{120} (3 - 2\sqrt{2})$

6.

 $\int_0^{\pi/2} \int_{1-\cos \theta}^{1+\sin \theta} r \, dr \, d\theta = \frac{1+2\pi}{4}$

7.

 $\int_0^{\pi/4} \int_{1-\cos 2\theta}^{1+\cos 2\theta} r \, dr \, d\theta = 1$

8.

 $\int_0^{\pi/2} \int_{1-\cos \theta}^{1+\sin \theta} \theta \, dr \, d\theta = \frac{\pi}{2}$

9.

 $\int_{\pi/4}^{3\pi/4} \int_{\operatorname{cosec} \theta}^{2\sin \theta} r \, dr \, d\theta = \frac{\pi}{2}$

10.

$$\textcircled{\text{P}} \int_{-\pi/2}^{\pi/2} \int_1^{1+\cos\theta} r \, dr \, d\theta = 2 + \frac{\pi}{4}$$

11.

$$\textcircled{\text{P}} \int_{-1}^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx = \frac{2}{3}$$

12.

$$\textcircled{\text{P}} \int_{-2}^2 \int_0^{\sqrt{4-y^2}} x \, dx \, dy = \frac{16}{3}$$

13.

$$\textcircled{\text{P}} \int_0^2 \int_0^{\sqrt{2x-x^2}} y^2 \, dy \, dx = \frac{\pi}{8}$$

14.

$$\textcircled{\text{P}} \int_0^4 \int_0^{\sqrt{4y-y^2}} xy \, dx \, dy = \frac{32}{3}$$

15.

$$\textcircled{\text{P}} \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dy \, dx = 4\pi$$

16.

$$\textcircled{\text{P}} \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} xy \, dy \, dx = 0$$

17.

$$\textcircled{\text{P}} \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2 + y^2) \, dx \, dy = \frac{\pi}{2}$$

18.

$$\textcircled{\text{P}} \int_{-1}^1 \int_0^{\sqrt{1-x^2}} e^{\sqrt{x^2+y^2}} \, dy \, dx = \pi$$

19.

$$\textcircled{\text{P}} \int_{-2}^2 \int_0^{\sqrt{4-x^2}} \sin(x^2 + y^2) \, dy \, dx = \frac{\pi \sin 4}{2}$$

20.

 $\int_{-1}^0 \int_{-\sqrt{1-y^2}}^0 x^2 y \, dx \, dy = -\frac{1}{15}$

21.

 $A = 8 + \pi$

22.

 $A = \frac{\pi}{2}$

23.

 $A = 4$

24.

 $A = 8\pi$

25.

 $\int_0^4 \int_{y/2}^{(y/2)+1} \frac{2x-y}{2} \, dx \, dy = 2$

26.

 $\iint_R (x^2 + y^2) \, dA = \frac{ab\pi}{4}(a^2 + b^2)$

27.

 $\frac{3\pi}{2}$

28.

 $\frac{\sqrt{\pi}}{2}$

29.

 $k = 6$

Section 8.4 연습문제

1.

 $\iiint_R (1 + r + \sin \theta) dz dr d\theta = 24\pi$

2.

 $\iiint_R r z dr dz d\theta = 8\pi$

3.

 $\iiint_R (r^2 \sin \theta + 2z) dz dr d\theta = 8\pi + \frac{32}{3}$

4.

 $\iiint_R \rho^2 \sin \phi \sin \theta d\rho d\phi d\theta = \frac{5}{4}$

5.

 $\iiint_R 3\rho^2 \sin \phi d\rho d\phi d\theta = \frac{5\pi}{2}$

6.

 $\iiint_R (x^2 + y^2) dV = \frac{2}{5}$

7.

 $\iiint_R \sqrt{x^2 + y^2} dV = 48\pi$

8.

 $\iiint_R x dV = \frac{\pi}{4}$

9.

 $V = 4\pi$

10.

 $V = 24\pi$

11.

 $V = \frac{5\pi}{4}$

12.

 $V = \frac{16(3\pi - 4)}{9}$

13.

 $V = \frac{\pi}{2}$

14.

 $V = 13\pi$

15.

 $\iiint_R dV = \frac{\pi}{8}$

16.

 $\iiint_R dV = \frac{148\pi}{3}$

17.

 $\iiint_R dV = 18\pi$

18.

 $\iiint_R dV = \frac{(144 - \pi)\pi}{8}$

19.

 $\iiint_R dV = \frac{175\pi}{96}$

20.

 $\iiint_R dV = 18\pi$