

Section 9.1 연습문제

1.

 $\|\mathbf{a}\| = \sqrt{5}$

2.

 $\|\mathbf{a}\| = 3\sqrt{2}$

3.

 $\|\mathbf{a}\| = 3\sqrt{2}$

4.

 $\|\mathbf{a}\| = 2\sqrt{3}$

5.

 $\mathbf{a} + \mathbf{b} = (-1, 4, 1)$

6.

 $2\mathbf{a} - \mathbf{b} = (-11, 11, 5)$

7.

 $\|-\mathbf{a} + 2\mathbf{b}\| = \sqrt{165}$

8.

 $\mathbf{u} = \frac{1}{\sqrt{6}}(1, 2, -1)$

9.

 $\mathbf{u} = \frac{1}{\sqrt{14}}(1, -3, -2)$

10.

 $\mathbf{u} = \frac{1}{\sqrt{6}}(1, -2, 1)$

11.

 $\mathbf{u} = \frac{1}{\sqrt{26}}(3, -1, 4)$

12.

$\mathbf{a} \cdot \mathbf{b} = -4, \theta = \cos^{-1}\left(-\frac{4}{5}\right)$

13.

$\mathbf{a} \cdot \mathbf{b} = 1, \theta = \cos^{-1}\left(\frac{4}{21}\right)$

14.

$\mathbf{a} \cdot \mathbf{b} = -6, \theta = \cos^{-1}\left(-\frac{\sqrt{6}}{3}\right)$

15.

$\mathbf{a} \cdot \mathbf{b} = -3, \theta = \cos^{-1}\left(-\frac{\sqrt{21}}{14}\right)$

16.

$\mathbf{a} \cdot \mathbf{b}$ 는 스칼라이므로 수학적으로 의미가 없다.

17.

$\mathbf{a} \cdot \mathbf{b}$ 는 스칼라이므로 스칼라와 벡터의 내적은 정의되지 않는다.

18.

$\mathbf{a} \cdot \mathbf{b}$ 는 스칼라이므로 스칼라와 벡터의 합은 정의되지 않는다.

19.

$\mathbf{a} \times \mathbf{b}$ 는 벡터이고 따라서 $(\mathbf{a} \times \mathbf{b}) + \mathbf{c}$ 은 $\mathbf{a}$ 와 $\mathbf{b}$ 의 외적인 벡터와 벡터 $\mathbf{c}$ 의 합이다.

20.

$\theta = \cos^{-1} \frac{1}{\sqrt{3}} \simeq 54.7^\circ$

21.

$a = \sqrt{2} \pm 1$

22.

$a = -\sqrt{2} \pm \sqrt[4]{2}$

23.

 $\mathbf{a} \times \mathbf{b} = (2, 6, 2)$

24.

 $\mathbf{a} \times \mathbf{b} = (5, 6, 7)$

25.

 $\mathbf{a} \times \mathbf{b} = (10, 1, -4)$

26.

 $\mathbf{a} \times \mathbf{b} = (6, 2, -2)$

27.



(a) $\mathbf{a} \times \mathbf{b} = (1, 2, -5)$

(b) $\mathbf{b} \times \mathbf{a} = (-1, -2, 5)$

(c) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (-4, 0, 4)$

(d) $\mathbf{b} \times \mathbf{c} = (2, 4, -2), \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 8$

(e) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (-8, 4, 0)$

(f) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{c}) = (16, -8, 0)$

28.

 $V = 16$

29.

 $V = 5$

30.

 $S = \sqrt{101}$

31.

 $S = \sqrt{291}$

32.

 $3\sqrt{19}$

33.

 $\sqrt{66}$

34.

 $W = 2500\sqrt{2}$ (joule)

35.

 $\|\tau\| = 28.977$ (joule)

36.

 [정리 9-3]의 (2)

$\mathbf{a} = (a_1, a_2, \dots, a_n)$, $\mathbf{b} = (b_1, b_2, \dots, b_n)$ 에 대하여

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + \dots + a_nb_n = b_1a_1 + b_2a_2 + \dots + b_na_n = \mathbf{b} \cdot \mathbf{a}$$

[정리 9-3]의 (3)

$\mathbf{a} = (a_1, a_2, \dots, a_n)$, $\mathbf{b} = (b_1, b_2, \dots, b_n)$, $\mathbf{c} = (c_1, c_2, \dots, c_n)$ 에 대하여

$\mathbf{b} + \mathbf{c} = (b_1 + c_1, b_2 + c_2, \dots, b_n + c_n)$ 이므로

$$\begin{aligned} \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) &= (a_1, a_2, \dots, a_n) \cdot (b_1 + c_1, b_2 + c_2, \dots, b_n + c_n) \\ &= (a_1b_1 + a_1c_1) + (a_2b_2 + a_2c_2) + \dots + (a_nb_n + a_nc_n) \\ &= (a_1b_1 + a_2b_2 + \dots + a_nb_n) + (a_1c_1 + a_2c_2 + \dots + a_nc_n) \\ &= \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} \end{aligned}$$

37.

 $\|\mathbf{a} + \mathbf{b}\|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b}$

$$= \|\mathbf{a}\|^2 + 2\mathbf{a} \cdot \mathbf{b} + \|\mathbf{b}\|^2$$

$$\begin{aligned} \|\mathbf{a} - \mathbf{b}\|^2 &= (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} - \mathbf{b} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b} \\ &= \|\mathbf{a}\|^2 - 2\mathbf{a} \cdot \mathbf{b} + \|\mathbf{b}\|^2 \end{aligned}$$

이므로 $\|\mathbf{a} + \mathbf{b}\|^2 + \|\mathbf{a} - \mathbf{b}\|^2 = 2(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2)$

38.

 $\mathbf{a} = (a_1, a_2, a_3)$, $\mathbf{b} = (b_1, b_2, b_3)$, $\mathbf{c} = (c_1, c_2, c_3)$ 에 대하여

$\mathbf{b} + \mathbf{c} = (b_1 + c_1, b_2 + c_2, b_3 + c_3)$ 이므로

$$\begin{aligned}
\mathbf{a} \times (\mathbf{b} + \mathbf{c}) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix} \\
&= (a_2 b_3 - a_3 b_2 + a_2 c_3 - a_3 c_2) \mathbf{i} + (a_3 b_1 - a_1 b_3 + a_3 c_1 - a_1 c_3) \mathbf{j} \\
&\quad + (a_1 b_2 - a_2 b_1 + a_1 c_2 - a_2 c_1) \mathbf{k} \\
&= (a_2 b_3 - a_3 b_2) \mathbf{i} + (a_3 b_1 - a_1 b_3) \mathbf{j} + (a_1 b_2 - a_2 b_1) \mathbf{k} \\
&\quad + (a_2 c_3 - a_3 c_2) \mathbf{i} + (a_3 c_1 - a_1 c_3) \mathbf{j} + (a_1 c_2 - a_2 c_1) \mathbf{k} \\
&= \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}
\end{aligned}$$

Section 9.2 연습문제

1.

 $a = 3, b = 2, c = d = 1$

2.

 $a = \frac{3}{2}, b = -\frac{1}{2}, c = 7, d = 3$

3.



(a) $A + 2B = \begin{pmatrix} 5 & 4 & -2 \\ -1 & 3 & 3 \\ 6 & 7 & 5 \end{pmatrix}$

(b) $A - B = \begin{pmatrix} -1 & 1 & -5 \\ 2 & 0 & 0 \\ -6 & 1 & -4 \end{pmatrix}$

(c) $A^t + B^t = \begin{pmatrix} 3 & 0 & 2 \\ 3 & 2 & 5 \\ -3 & 2 & 2 \end{pmatrix}$

(d) $A + A^t = \begin{pmatrix} 2 & 3 & -6 \\ 3 & 2 & 4 \\ -6 & 4 & -2 \end{pmatrix}$

(e) $A - A^t = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 2 & 2 & 0 \end{pmatrix}$

(f) $AB = \begin{pmatrix} -16 & -5 & -9 \\ 5 & 4 & 5 \\ -11 & -1 & -2 \end{pmatrix}, (AB)^t = \begin{pmatrix} -16 & 5 & -11 \\ -5 & 4 & -1 \\ -9 & 5 & -2 \end{pmatrix}$

(g) $A^t = \begin{pmatrix} 1 & 1 & -2 \\ 2 & 1 & 3 \\ -4 & 1 & -1 \end{pmatrix}, A^t B = \begin{pmatrix} -7 & -2 & -4 \\ 15 & 9 & 12 \\ -13 & -5 & -6 \end{pmatrix}$

(h) $B^t = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}, BB^t = \begin{pmatrix} 6 & 0 & 13 \\ 0 & 3 & 1 \\ 13 & 1 & 29 \end{pmatrix}$

4.



$A^t = \begin{pmatrix} 1 & 3 & 2 \\ 5 & 1 & 0 \\ 2 & 9 & 3 \end{pmatrix}$ 이므로 대칭행렬과 교대행렬은 각각 다음과 같다.

$$\frac{1}{2}(A + A^t) = \begin{pmatrix} 1 & 4 & 2 \\ 4 & 1 & 9/2 \\ 2 & 9/2 & 3 \end{pmatrix}, \quad \frac{1}{2}(A - A^t) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 9/2 \\ 0 & -9/2 & 0 \end{pmatrix}$$

5.

$$\textcircled{\text{II}} k = -\frac{2}{7}$$

6.

$$\textcircled{\text{II}} k = \frac{-2 \pm \sqrt{6}}{2}$$

7.

$$\textcircled{\text{II}} f(A) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

8.

$$\textcircled{\text{II}} f(A) = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 2 & -2 & 2 & 0 \\ 0 & 2 & -2 & 2 \\ 0 & 2 & 2 & -2 \end{pmatrix}$$

9.

$$\textcircled{\text{II}} A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{pmatrix}$$

10.

$$\textcircled{\text{II}} A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

11.

$$\textcircled{\text{II}} A = \begin{pmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$$

12.

$$\textcircled{\text{II}} A = \begin{pmatrix} 0 & -1 & -2 & -1 \\ 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$

13.

 $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

14.

 임의의 실수 x, y, z 에 대하여 $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ 0 \\ 0 \end{pmatrix}$ 를 만족하는 행렬 A 가 존재한다고 하자. 그

러면 $A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 이다. 한편 $A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = -A \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 이어야 한다. 따라서

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ 이므로 $1 = -1$ 이라는 모순을 얻는다. 따라서 주어진 식을 만족하는

3×3 행렬 A 는 존재하지 않는다.

15.

 A, B, C, D가 각각 수학, 물리, 화학 교재를 산 가격을 의미한다.

Section 9.3 연습문제

1.

$$\textcircled{\text{문제}} \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}^{-1} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix}$$

2.

$$\textcircled{\text{문제}} \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{pmatrix}$$

3

$$\textcircled{\text{문제}} \begin{pmatrix} 2 & 1 & -3 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{11} & \frac{1}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{7}{11} & -\frac{12}{11} \\ -\frac{2}{1} & \frac{3}{1} & -\frac{2}{11} \end{pmatrix}$$

4.

$$\textcircled{\text{문제}} \begin{pmatrix} 3 & 2 & -1 \\ 1 & 4 & 1 \\ -1 & 0 & 4 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{8}{17} & -\frac{4}{17} & \frac{3}{17} \\ -\frac{5}{34} & \frac{11}{34} & -\frac{2}{17} \\ \frac{2}{17} & -\frac{1}{17} & \frac{5}{17} \end{pmatrix}$$

5.

$$\textcircled{\text{문제}} \begin{pmatrix} -3 & 4 & 1 & -3 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & -1 & 1 \\ 2 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ -\frac{5}{12} & \frac{1}{12} & -\frac{3}{2} & \frac{1}{12} \\ -\frac{1}{2} & \frac{1}{2} & -1 & -\frac{1}{2} \end{pmatrix}$$

6.

$$\textcircled{\text{문제}} \begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{4}{3} & -\frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & -\frac{1}{6} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$

7.

$$\textcircled{\text{필요}} \quad a \neq 0, \quad b \neq 0, \quad c \neq 0, \quad d \neq 0, \quad \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 1 & 0 & 0 & d \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{a} & 0 & 0 & 0 \\ 0 & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{1}{c} & 0 \\ -\frac{1}{ad} & 0 & 0 & \frac{1}{d} \end{pmatrix}$$

8.

$$\textcircled{\text{필요}} \quad a \neq 0, \quad b \neq 0, \quad c \neq 0, \quad d \neq 0, \quad \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{d} \\ 0 & 0 & \frac{1}{c} & 0 \\ 0 & \frac{1}{b} & 0 & 0 \\ \frac{1}{a} & 0 & 0 & 0 \end{pmatrix}$$

9.

$$\textcircled{\text{필요}} \quad a \neq 0, \quad \begin{pmatrix} a & 0 & 0 & 0 \\ 1 & a & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & a \end{pmatrix}^{-1} = \frac{1}{a^4} \begin{pmatrix} a^3 & 0 & 0 & 0 \\ -a^2 & a^3 & 0 & 0 \\ a & -a^2 & a^3 & 0 \\ -1 & a & -a^2 & a^3 \end{pmatrix}$$

10.

$$\textcircled{\text{필요}} \quad a \neq \frac{-1 \pm \sqrt{5}}{2}, \quad \frac{1 \pm \sqrt{5}}{2}$$

$$\begin{pmatrix} a & 1 & 0 & 0 \\ 1 & a & 1 & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & 1 & a \end{pmatrix}^{-1} = \frac{1}{a^4 - 3a^2 + 1} \begin{pmatrix} a^3 - 2a & 1 - a^2 & a & -1 \\ 1 - a^2 & a^3 - a & -a^2 & a \\ a & -a^2 & a^3 - a & 1 - a^2 \\ -1 & a & 1 - a^2 & a^3 - 2a \end{pmatrix}$$

11.

$$\textcircled{\text{필요}} \quad \begin{pmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \\ -1 & 2 & 3 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & \frac{148}{29} \\ 0 & 1 & 0 & -\frac{6}{29} \\ 0 & 0 & 1 & \frac{34}{29} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

12.

$$\textcircled{\text{풀이}} \begin{pmatrix} 1 & -1 & 3 & 2 \\ -2 & 1 & 5 & 1 \\ -3 & 2 & 2 & -1 \\ 4 & -3 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -8 & -3 \\ 0 & 1 & -11 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

13.

$$\textcircled{\text{풀이}} \begin{pmatrix} 3 & 1 & 2 & -3 \\ 1 & -4 & 1 & 2 \\ 1 & -2 & 2 & 1 \\ -2 & 2 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

14.

$$\textcircled{\text{풀이}} \begin{pmatrix} 3 & 0 & 2 & 3 & 2 \\ -3 & 2 & 2 & -1 & 2 \\ 2 & -3 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{7}{9} \\ 0 & 1 & 0 & 0 & \frac{7}{45} \\ 0 & 0 & 1 & 0 & \frac{16}{15} \\ 0 & 0 & 0 & 1 & -\frac{37}{45} \end{pmatrix}$$

15.

$$\textcircled{\text{풀이}} b_1 = 2b_2$$

16.

$$\textcircled{\text{풀이}} b_1 = b_2 + b_3$$

17.

$$\textcircled{\text{풀이}} b_2 = 2b_1 + b_3$$

18.

$$\textcircled{\text{풀이}} x = 12, y = 2, z = -3$$

19.

$$\textcircled{\text{풀이}} x = 3, y = -4, z = -\frac{8}{3}, w = \frac{2}{3}$$

20

$$\textcircled{\text{풀이}} x = \frac{7}{4} + \frac{9}{8}t, y = -\frac{11}{2} - \frac{15}{4}t, z = -\frac{11}{2} - \frac{25}{8}t, w = t$$

21.

 $x = \pm 1, y = \pm \sqrt{3}, z = \pm \sqrt{2}$

22

 $i_1 = \frac{13}{5}, i_2 = -\frac{2}{5}, i_3 = \frac{11}{5}$

23

 $i_1 = -\frac{5}{22}, i_2 = \frac{7}{22}, i_3 = \frac{6}{11}$

Section 9.4 연습문제

1.

$$\text{풀이} \quad \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = -11$$

2.

$$\text{풀이} \quad \begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 6$$

3.

$$\text{풀이} \quad \begin{vmatrix} 1 & 2 & 4 & 1 \\ 2 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 1 \end{vmatrix} = 0$$

4.

$$\text{풀이} \quad \begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} = 4ab$$

5.

$$\text{풀이} \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -a^3 - b^3 - c^3 + 3abc$$

6.

$$\text{풀이} \quad \begin{vmatrix} a & 0 & c & 0 \\ 0 & a & 0 & c \\ b & 0 & d & 0 \\ 0 & b & 0 & d \end{vmatrix} = (bc - ad)^2$$

7.

$$\text{풀이} \quad \begin{vmatrix} 1 & a & c \\ 1 & b & a \\ 1 & c & b \end{vmatrix} = a^2 + b^2 + c^2 - ab - bc - ca$$

8.

$$\text{풀이} \quad \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

9.

$$\textcircled{\text{풀이}} \begin{vmatrix} b^2 + c^2 & ab & ca \\ ab & c^2 + a^2 & ca \\ ca & bc & a^2 + b^2 \end{vmatrix} = bc^2(a^3 + 3a^2b - ab^2 + b^3 - ac^2 + bc^2)$$

10.

$$\textcircled{\text{풀이}} \begin{vmatrix} 1 & a & a^2 \\ 1 & a & a^2 \\ 1 & a & a^2 \end{vmatrix} = 0$$

11.

$$\textcircled{\text{풀이}} \begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3$$

12.

$$\textcircled{\text{풀이}} \begin{vmatrix} \sin \theta & \cos \theta & 0 \\ -\cos \theta & \sin \theta & 0 \\ \sin \theta - \cos \theta & \sin \theta + \cos \theta & 1 \end{vmatrix} = 1$$

13.

$$\textcircled{\text{풀이}} \lambda = -1 \pm \sqrt{13}$$

14.

$$\textcircled{\text{풀이}} \lambda = 1$$

15.

$$\textcircled{\text{풀이}} x = -1$$

16.

$$\textcircled{\text{풀이}} x = 3$$

17.

$$\textcircled{\text{풀이}} \text{두 점 } (a_1, b_1) \text{과 } (a_2, b_2) \text{를 지나는 직선 } y = \frac{b_2 - b_1}{a_2 - a_1}x + \frac{a_2b_1 - a_1b_2}{a_2 - a_1} \text{ 이다.}$$

18.

$\textcircled{\text{풀이}}$ 성질 (6)을 이용하여 3열에 있는 각각의 성분을 3열로 가지는 세 행렬의 합으로 바꾸면 성질 (8)에 의하여 성립한다.

19.

풀이 성질 (6)에 의하여 2열을 1열에 더하고 성질 (9)에 의하여 1열 성분에서 인수 2를 행렬식 밖으로 꺼낸다. 성질 (6)에 의하여 1열에 (-1) 을 곱하여 2열에 더한다. 성질 (9)에 의하여 1열 성분에서 인수 -1 을 행렬식 밖으로 꺼낸다.

20.

풀이 성질 (6)에 의하여 1행에 $-t$ 를 곱하여 2행에 더한 후, 2행의 인수 $1-t^2$ 을 행렬식 밖으로 꺼낸다. 2행에 $-t$ 를 곱하여 1행에 더한다.

21.

풀이 1열에 $-t$ 를 곱하여 2열에 더한다. 1열에 $-r$ 을 곱하여 3열에 더하고, 다시 2열에 $-s$ 를 곱하여 3열에 더한다.

22.

풀이

$$\text{3행에 관한 여인수 전개 : } \begin{vmatrix} 2 & 1 & 0 & 3 \\ -1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 1 & 1 \end{vmatrix} = -7$$

$$\text{3열에 관한 여인수 전개 : } \begin{vmatrix} 2 & 1 & 0 & 3 \\ -1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 3 & 2 & 1 & 1 \end{vmatrix} = -7$$

23.

풀이

(a) $|2A| = 32$

(b) $|2A^{-1}| = 2$

(c) $|2A^t| = 8$

(d) $|(2A)^{-2}| = \frac{1}{1024}$

24.

풀이 $\begin{vmatrix} \sin^2 \alpha & \sin^2 \beta & \sin^2 \gamma \\ \cos^2 \alpha & \cos^2 \beta & \cos^2 \gamma \\ 1 & 1 & 1 \end{vmatrix} = 0$ 이므로 역행렬을 갖지 않는다.

25.

$$\textcircled{\text{풀이}} A^{-1} = \frac{1}{10} \begin{pmatrix} 8 & -4 & 2 & -4 \\ -2 & 6 & -3 & 1 \\ -4 & 2 & 4 & 2 \\ -4 & 2 & -1 & 7 \end{pmatrix}$$

26.

$$\textcircled{\text{풀이}} x_1 = \frac{-84}{-14} = 6, \quad x_2 = \frac{-70}{-14} = 5, \quad x_3 = \frac{0}{-14} = 0$$

27.

$$\textcircled{\text{풀이}} x_1 = \frac{474}{-140} = -\frac{237}{70}, \quad x_2 = \frac{360}{-140} = -\frac{18}{7}, \quad x_3 = \frac{-662}{-140} = \frac{331}{70}, \quad x_4 = \frac{-890}{-140} = \frac{89}{14}$$

28.

$$\textcircled{\text{풀이}} i_1 = \frac{|A_1|}{|A|} = 6, \quad i_2 = \frac{|A_2|}{|A|} = -5, \quad i_3 = \frac{|A_3|}{|A|} = 1$$

29.

$$\textcircled{\text{풀이}} i_1 = \frac{|A_1|}{|A|} = 1, \quad i_2 = i_3 = \frac{|A_2|}{|A|} = 1, \quad i_4 = i_6 = \frac{|A_4|}{|A|} = 0, \quad i_5 = \frac{|A_5|}{|A|} = -1$$