

CHAPTER 12

12.1

(a) $\sqrt{3} + i$

(b) $\frac{\sqrt{2}}{2} + i\frac{\sqrt{6}}{2}$

(c) $2\sqrt{2} + i2\sqrt{2}$

(d) -3

12.2

(a) $|z| = \sqrt{5}$, $\theta = \tan^{-1}2$, $z = |z| \angle \theta$

(b) $z = 2 \angle \frac{\pi}{2}$

(c) $z = 1 \angle \frac{3}{2}\pi$

(d) $|z| = \frac{\sqrt{5}}{4}$, $\theta = \tan^{-1}2$, $z = |z| \angle \theta$

12.3

(a) $5 - 2i$

(b) $1 + 5i$

(c) $3 - i$

(d) $\frac{2}{3} + \frac{1}{3}i$

12.4

(a) $-8 + 8\sqrt{3}i$

(b) $8i$

(c) 4^6

(d) $32i$

12.5

(a) $(\sqrt{3} - i)^{\frac{1}{3}} = 2^{\frac{1}{3}}(\cos\frac{\pi}{18} - i\sin\frac{\pi}{18})$, $2^{\frac{1}{3}}(\cos\frac{13}{18}\pi - i\sin\frac{13}{18}\pi)$, $2^{\frac{1}{3}}(\cos\frac{25}{18}\pi - i\sin\frac{25}{18}\pi)$

(b) $(1 + i)^{\frac{1}{5}} = \sqrt[5]{2^{\frac{1}{5}}} = \left[\cos\frac{1}{5}(\frac{\pi}{4} + 2k\pi) + i\sin\frac{1}{5}(\frac{\pi}{4} + 2k\pi) \right]$, $k = 0, 1, 2, 3, 4$

$$(c) \quad (2 + 2\sqrt{3}i)^{\frac{1}{4}} = 4^{\frac{1}{4}} \left[\cos \frac{1}{4} \left(\frac{\pi}{3} + 2\pi k \right) + i \sin \frac{1}{4} \left(\frac{\pi}{3} + 2\pi k \right) \right], \quad k = 0, 1, 2, 3$$

$$(d) \quad (1 - i)^{\frac{1}{5}} = \sqrt[5]{2^{\frac{1}{5}}} \left[\cos \frac{1}{5} \left(\frac{\pi}{4} + 2\pi k \right) - i \sin \frac{1}{5} \left(\frac{\pi}{4} + 2\pi k \right) \right], \quad k = 0, 1, 2, 3, 4$$

12.6

$$(a) \quad \lim_{z \rightarrow i} (2z^2 - 3z + 1) = -1 - 3i$$

$$(b) \quad \lim_{z \rightarrow i} \frac{z^4 - 1}{z - i} = 0$$

12.7

$$(a) \quad f'(z) = 4z - 3$$

$$(b) \quad f'(z) = (z - 1)(2z - 2) + (1)(z^2 - 2z + 3)$$

$$(c) \quad f'(z) = \frac{-z^2 + 4z + 1}{(z^2 + 1)^2}$$

$$(d) \quad f'(z) = \frac{(4z + 3)(z^2 + 1) - (z^2 + 3z - 1)2z}{(z^2 + 1)^2}$$

12.8

(a) 해석적이지 않다.

(b) 해석적이지 않다.

(c) $z = 1$ 을 제외한 모든 점에서 해석적이다.

(d) 해석적이다.

12.9

$$(a) \quad \ln z = \ln 2 + i(\pi \pm 2\pi n), \quad n = 0, 1, 2, \dots$$

$$\operatorname{Ln} z = \ln 2 + i\pi$$

$$(b) \quad \ln(\sqrt{3} + i) = \ln 2 + i \left(\frac{\pi}{6} \pm 2\pi n \right)$$

$$\operatorname{Ln}(\sqrt{3} + i) = \ln 2 + \frac{\pi}{6}i$$

$$(c) \ln z = \ln \sqrt{2} + i \left(-\frac{\pi}{4} \pm 2n\pi \right), \quad n = 0, 1, 2, \dots$$

$$\operatorname{Ln} z = \ln \sqrt{2} - i \frac{\pi}{4}$$

$$(d) \ln z = \ln \sqrt{2} + i \left(\frac{\pi}{4} \pm 2n\pi \right), \quad n = 0, 1, 2, \dots$$

$$\operatorname{Ln} z = \ln \sqrt{2} + i \frac{\pi}{4}$$

12.10

$$(a) \ln(z_1 z_2) = \ln 4 \sqrt{2} + i \left(-\frac{\pi}{4} \pm 2n\pi \right), \quad n = 1, 2, 3, \dots$$

$$\operatorname{Ln}(z_1 z_2) = \ln 4 \sqrt{2} - i \frac{\pi}{4}$$

$$(b) \ln(z_1 z_2) = \ln(2 \sqrt{2}) + i \left(-\frac{\pi}{12} \pm 2\pi n \right), \quad n = 0, 1, 2, \dots$$

$$\operatorname{Ln}(z_1 z_2) = \ln(2 \sqrt{2}) - i \frac{\pi}{12}$$

12.11

$$(a) -\frac{52}{3} - \frac{64}{3}i$$

$$(b) -\frac{4}{3} + \frac{4}{3}i$$

$$(c) -i\pi$$

$$(d) i\pi$$

12.12

$$(a) \int_C (z^2) dz = 0$$

$$(b) \int_C (2z^2 - 1) dz = 0$$

12.13

$$(a) \int_{C_1} f(z) dz = \frac{(1+i)^3}{3}$$

$$(b) \int_{C_2} f(z) dz = \frac{(1+i)^3}{3}$$

$$(c) \int_C f(z) dz = 0$$

12.14

$$(a) \int_C 2z dz = 2i - 1$$

$$(b) \int_C 2z dz = 2i - 1$$

$$(c) \int_C 2z dz = 2i - 1$$

12.15

$$(a) \int_C (3z - 1) dz = -\frac{11}{2} + 4i$$

$$(b) \int_C e^z dz = e^{1+2i} - 1$$

12.16

$$\int_C \frac{1}{z-a} dz = 2\pi i$$

12.17

$$\int_C \frac{e^z}{z-2} dz = e^2 (2\pi i)$$

12.18

$$(a) -\pi i (\cos 2)$$

$$(b) 2\pi i \left(-\frac{1}{9} \sin 1 - \frac{1}{9} \sin 2 + \frac{1}{3} \cos 2 \right)$$

12.19

$$\operatorname{Res}\left(e^{-\frac{1}{z^2}}, 0\right)=0$$

12.20

$$f(z)=\frac{3}{z-2}+\frac{1}{6}-\frac{1}{6^2}(z-2)+\frac{1}{6^3}(z-2)^2-\cdots, \quad 0<|z-2|<6$$

12.21

$$\int_C \frac{\cos z}{z} dz=2\pi i$$