

CHAPTER 03

3.1

$$(a) \quad \mathcal{L} \{5t^2 + 2\} = 5 \frac{2}{s^3} + 2 \frac{1}{s}$$

$$(c) \quad \mathcal{L} \{\cos 2t\} = \frac{s}{s^2 + 4}$$

$$(e) \quad \mathcal{L} \{te^{-2t}\} = \frac{1}{(s+2)^2}$$

$$(g) \quad \mathcal{L} \{2t \sin t\} = \frac{4s}{(s^2 + 1)^2}$$

$$(i) \quad \mathcal{L} \{f(t)\} = -\frac{4}{s}e^{-2s} + \frac{2}{s}$$

$$(k) \quad \mathcal{L} \{f(t)\} = \frac{1}{s+1} [1 - e^{-(s+1)}]$$

$$(m) \quad \mathcal{L} \{f(t)\} = -\frac{2}{s} [e^{-s} - 1]$$

$$(b) \quad \mathcal{L} \{e^{2t-2}\} = \frac{e^{-2}}{s-2}$$

$$(d) \quad \mathcal{L} \{\sin 3t\} = \frac{3}{s^2 + 9}$$

$$(f) \quad \mathcal{L} \{t^2 e^{2t}\} = \frac{2}{(s-2)^3}$$

$$(h) \quad \mathcal{L} \{t \cos 2t\} = -\frac{1(s^2 + 2^2) - s \cdot 2s}{(s^2 + 2^2)^2}$$

$$(j) \quad \mathcal{L} \{f(t)\} = -\frac{1}{s} [e^{-s} - 1]$$

$$(l) \quad \mathcal{L} \{f(t)\} = \frac{2}{-(s+1)} [e^{-(s+1)} - 1]$$

$$(n) \quad \mathcal{L} \{f(t)\} = \frac{e^{-s}}{s^2}$$

3.2

$$(a) \quad \mathcal{L} [f(t)] = \frac{1}{1 - e^{-2s}} \left(\frac{1}{s} - 2 \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} \right)$$

$$(b) \quad \mathcal{L} \{f(t)\} = \frac{1}{1 - e^{-2s}} \left\{ -\frac{1}{s} e^{-s} - \frac{1}{s^2} [e^{-s} - 1] \right\}$$

3.3

$$(a) \quad \mathcal{L} [e^{-2t} t] = \frac{1}{(s+2)^2}$$

$$(c) \quad \mathcal{L} \{f(t)\} = \frac{s+3}{(s+1)^2 + 2^2}$$

$$(e) \quad \mathcal{L} [(t-1)u(t-1)] = \frac{e^{-s}}{s^2}$$

$$(g) \quad \mathcal{L} \{e^{2t} u(t-1)\} = \frac{e^{-(s-2)}}{s-2}$$

$$(b) \quad \mathcal{L} [e^{-2t} \sin t] = \frac{1}{(s+2)^2 + 1}$$

$$(d) \quad \mathcal{L} \{e^{-2t} \sin(2t + \theta)\} = \frac{2 \cos \theta + (s+2) \sin \theta}{(s+2)^2 + 2^2}$$

$$(f) \quad \mathcal{L} [tu(t-3)] = \frac{e^{-3s}}{s^2} + \frac{3e^{-3s}}{s}$$

$$(h) \quad \mathcal{L} \{u(t-\pi) \cos 2t\} = e^{-\pi s} \frac{s}{s^2 + 2^2}$$

3.4

$$(a) \quad \mathcal{L} [f(t)] = \frac{1}{(s+2)^2}$$

$$(c) \quad \mathcal{L} [f(t)] = \frac{s^2 - 4}{(s^2 + 4)^2}$$

$$(b) \quad \mathcal{L} [f(t)] = \frac{2}{(s-2)^3}$$

$$(d) \quad \mathcal{L} [f(t)] = \frac{4(3s^2 - 4)}{(s^2 + 4)^3}$$

$$(e) \quad \mathcal{L}[f(t)] = \frac{4(s+1)}{[(s+1)^2 + 2^2]^2}$$

$$(f) \quad \mathcal{L}[f(t)] = \frac{s^2 - 2s - 3}{[(s-1)^2 + 2^2]^2}$$

3.5

$$(a) \quad \mathcal{L}[f(t)] = \frac{4}{s^4}$$

$$(b) \quad \mathcal{L}[f(t)] = \frac{s}{(s+1)(s^2+1)}$$

$$(c) \quad \mathcal{L}[f(t)] = \frac{1}{s^2-1}$$

$$(d) \quad \mathcal{L}[f(t)] = \frac{2}{s^3-2s^2}$$

$$(e) \quad \mathcal{L}[f(t)] = \frac{s}{(s^2+1)^2}$$

$$(f) \quad \mathcal{L}[f(t)] = \frac{1}{s^2+1}$$

3.6

$$(a) \quad \mathcal{L}[f(t)] = \frac{1}{s(s+1)}$$

$$(b) \quad \mathcal{L}[f(t)] = \frac{2}{s^3+4s}$$

3.7

$$(a) \quad \mathcal{L}^{-1}[F(s)] = -\frac{1}{2}e^{-t} + \frac{3}{2}e^{3t}$$

$$(b) \quad \mathcal{L}^{-1}[F(s)] = e^{2t}$$

$$(c) \quad \mathcal{L}^{-1}[F(s)] = \cos 2t + \frac{3}{2} \sin 2t$$

$$(d) \quad \mathcal{L}^{-1}[F(s)] = u(t) + 2t$$

$$(e) \quad \mathcal{L}^{-1}[F(s)] = -\frac{7}{4}e^{-t} - \frac{3}{2}te^{-t} + 2u(t) - \frac{1}{4}e^t$$

$$(f) \quad \mathcal{L}^{-1}[F(s)] = 2u(t) - t + e^{3t}$$

$$(g) \quad \mathcal{L}^{-1}[F(s)] = -\frac{2}{9}u(t) + \frac{1}{9}e^{3t} + \frac{1}{9}e^{-3t}$$

$$(h) \quad \mathcal{L}^{-1}[F(s)] = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^t$$

$$(i) \quad \mathcal{L}^{-1}[F(s)] = \frac{1}{2}e^t - \frac{1}{2}(\cos t + \sin t)$$

$$(j) \quad \mathcal{L}^{-1}[F(s)] = \cos 2t + \frac{1}{2} \sin 2t$$

3.8

$$(a) \quad \mathcal{L}^{-1}[F(s)] = \frac{1}{2}t^2e^{-t}$$

$$(b) \quad \mathcal{L}^{-1}[F(s)] = e^{-t} \cos t + e^{-t} \sin t$$

- (c) $\mathcal{L}^{-1}[F(s)] = e^{-2t} \cos 2t - e^{-2t} \sin 2t$
 (d) $\mathcal{L}^{-1}[F(s)] = e^{-t} \cos 2t + e^{-t} \sin 2t$
 (e) $\mathcal{L}^{-1}[F(s)] = 3e^{-2t} - 5te^{-2t}$
 (f) $\mathcal{L}^{-1}[F(s)] = 2t - 3u(t) + te^{-t} + 3e^{-t}$
 (g) $\mathcal{L}^{-1}[F(s)] = -u(t-2) + e^{t-2}u(t-2)$
 (h) $\mathcal{L}^{-1}[F(s)] = 2e^{-t} + e^{-(t-2)}u(t-2)$
 (i) $\mathcal{L}^{-1}[F(s)] = \frac{1}{2} \sin 2(t-2)u(t-2)$
 (j) $\mathcal{L}^{-1}[F(s)] = \frac{1}{4}e^{t-2}u(t-2) - \frac{1}{4}e^{-3(t-2)}u(t-2)$

3.9

- (a) $y(t) = \frac{2}{5}e^{2t} + \frac{1}{5} \sin t - \frac{2}{5} \cos t$
 (b) $y(t) = e^t$
 (c) $y(t) = -\frac{3}{2} + \frac{1}{4}(1 + \sqrt{2})e^{\sqrt{2}t} + \frac{1}{4}(1 - \sqrt{2})e^{-\sqrt{2}t}$
 (d) $y(t) = -\frac{1}{2}e^{-t} + \frac{7}{6}e^t + \frac{1}{3}e^{-2t}$
 (e) $y(t) = \frac{1}{6}e^{-t} + \frac{3}{2}e^t - \frac{2}{3}e^{2t}$
 (f) $y(t) = -\frac{2}{27}e^t + \frac{1}{9}te^t + \frac{2}{27}e^{-2t} + \frac{1}{9}te^{-2t}$
 (g) $y(t) = -\frac{3}{2}u(t) - t + \frac{5}{3}e^t - \frac{1}{6}e^{-2t}$

3.10

- (a) $y(t) = e^{2t} + te^{2t}$
 (b) $y(t) = te^{-t} + e^{-t}$
 (c) $y(t) = -e^t \cos 2t + e^t \sin 2t$
 (d) $y(t) = -\frac{1}{4}u(t) + \frac{1}{4}t + \frac{1}{4}e^{-2t} + \frac{1}{4}te^{-2t}$
 (e) $y(t) = \frac{1}{4}e^{-t} - \frac{1}{4}e^{-t} \cos 2t + \frac{1}{2}e^{-t} \sin 2t$
 (f) $y(t) = e^{-t} - e^{-2t} - te^{-2t}$
 (g) $y(t) = \frac{1}{5} \sin t - \frac{2}{5} \cos t + \frac{1}{5}e^{-t} \sin t + \frac{2}{5}e^{-t} \cos t$

3.11

$$(a) \ y(t) = -2u(t-2) + 2e^{t-2}u(t-2)$$

$$(b) \ y(t) = -\frac{1}{9}u(t) + \frac{1}{3}t + \frac{1}{9}e^{-3t} + \frac{1}{9}u(t-2) - \frac{1}{3}(t-2)u(t-2) + \frac{5}{9}e^{-3(t-2)}u(t-2)$$

$$(c) \ y(t) = -u(t) + \cos t + u(t-2) - \cos(t-2)u(t-2)$$

$$(d) \ y(t) = \frac{1}{3}e^{-t} + \frac{2}{3}e^{2t} - \frac{1}{2}u(t-1) + \frac{1}{3}e^{-(t-1)}u(t-1) + \frac{1}{6}e^{2(t-1)}u(t-1)$$

$$(e) \ y(t) = \frac{1}{5}e^{2t} - \frac{1}{5}e^{-3t} - \frac{1}{6}u(t-1) + \frac{1}{10}e^{2(t-1)}u(t-1) + \frac{1}{15}e^{-3(t-1)}u(t-1) \\ + \frac{1}{6}u(t-2) - \frac{1}{10}e^{2(t-2)}u(t-2) - \frac{1}{15}e^{-3(t-2)}u(t-2)$$

$$(f) \ y(t) = \sin t + \frac{1}{2}(t-\pi)\sin(t-\pi)u(t-\pi)$$

3.12

$$(a) \ y = -\frac{3}{25}\cos t - \frac{4}{25}\sin t + \frac{2}{5}t\cos t + \frac{1}{5}t\sin t + \frac{28}{25}e^{-2t}$$

$$(b) \ y = -\frac{1}{75}e^{-2t} + \frac{1}{12}e^t + \frac{1}{100}e^{3t}(10t-7)$$

3.13

$$x(t) = \frac{3}{2}u(t) - \frac{3}{2}\cos\sqrt{2}t - \frac{3}{2}u(t-2) + \frac{3}{2}\cos\sqrt{2}(t-2)u(t-2)$$

3.14

$$q(t) = \frac{1}{4}u(t-1) - \frac{1}{4}e^{-2(t-1)}u(t-1)$$

3.15

[예제 3-27]의 질량-스프링 시스템과 상사 시스템이므로 풀이 과정은 동일하다. 따라서 생략한다.

3.16

[예제 3-28]의 질량-스프링 시스템과 상사 시스템이므로 풀이 과정은 동일하다. 따라서 생략한다.

3.17

[예제 3-29]의 질량-댐퍼-스프링 시스템과 상사 시스템이므로 풀이 과정은 동일하다. 따라서 생략한다.

3.18

[예제 3-30]의 질량-댐퍼-스프링 시스템과 상사 시스템이므로 풀이과정은 동일하다. 따라서 생략한다.

3.19

(a) $x(t) = e^{-3t}$, $y(t) = -e^{-3t}$

(b) $x(t) = e^{3t}$, $y(t) = e^{3t}$

(c) $x(t) = -\frac{1}{4} + \frac{5}{4}e^{4t}$, $y(t) = -\frac{1}{4} + \frac{5}{4}e^{4t}$

(d) $x(t) = -\frac{2}{13}e^{-2t} + \frac{2}{13}e^t(\cos 2t + 5\sin 2t)$, $y(t) = -\frac{7}{13}e^{-2t} + \frac{4}{13}e^t(5\cos 2t - \sin 2t)$

3.20

(a) $\frac{Y(s)}{F(s)} = \frac{s+2}{(s+1)(s+3)}$

(b) 극점 : $s = -1, -3$, 영점 : $s = -2$ 를 [그림 3-26]과 같은 좌표평면상에 표시하면 된다.

(c) 시스템은 안정하다.