

CHAPTER 05

5.1

(a) $y = 2x^2$, 그래프는 포물선

(b) $x^2 + \frac{y^2}{2^2} = 1$, 그래프는 중심이 원점이고, 장축이 2, 단축이 1인 타원

(c) $x^2 + y^2 = 9$, $z = 2$, 그래프는 $z = 2$ 인 xy 평면상에서 중심이 원점이고, 반경이 3인 원

(d) $x = 2$, $y^2 + z^2 = 1$, 그래프는 $x = 2$ 인 yz 평면상에서 중심이 원점이고, 반경이 1인 원

5.2

(a) $\lim_{t \rightarrow 0} \vec{r}(t) = -\vec{j}$

(b) $\lim_{t \rightarrow 0} \vec{r}(t) = \vec{i} + 2\vec{j} - \frac{1}{2}\vec{k}$

(c) $\lim_{t \rightarrow 0} \vec{r}(t) = -\vec{j} + \vec{k}$

5.3

(a) $\lim_{t \rightarrow \infty} \vec{r}(t) = 2\vec{i}$

(b) $\lim_{t \rightarrow \infty} \vec{r}(t) = -\vec{k}$

(c) $\lim_{t \rightarrow \infty} \vec{r}(t) = -\vec{i} + 2\vec{j} - 2\vec{k}$

5.4

(a) $\vec{r}'(t) = (2t+2)\vec{i} - \frac{2}{(t-1)^2}\vec{j} - 4e^{-2t}\vec{k}$

$$\vec{r}'(0) = 2\vec{i} - 2\vec{j} - 4\vec{k}$$

(b) $\vec{r}'(t) = -2e^{-2t}(\sin 2t + \cos 2t)\vec{i} + 4t\vec{j} - 2e^{-2t}\vec{k}$

$$\vec{r}'(0) = -2\vec{i} - 2\vec{k}$$

5.5

(a) $-2e^{-t}\sin t$

(b) $2e^{-t}\cos t\vec{k}$

5.6

$$(a) \frac{d}{dt} [\vec{r}'(t) \times t\vec{r}(t)] = t\vec{r}''(t) \times \vec{r}(t)$$

$$(b) \frac{d}{dt} [\vec{r}'(t)] \times e^t \vec{r}(t) = e^t \vec{r}'(t) \times \vec{r}(t)$$

$$(c) \frac{d}{dt} [\vec{r}(t) \cdot \{\vec{r}(t) \times \vec{r}'(t)\}] = 0$$

$$(d) \frac{d}{dt} [\vec{r}'(t^2) + t\vec{r}(t^2)] = 2t\vec{r}''(t^2) + 2t^2\vec{r}'(t) + t\vec{r}(t^2)$$

5.7

$$(a) \vec{r}''(t) = e^{-2t}(3\cos t + 4\sin t)\vec{i} + 2\vec{j} + 4e^{-2t}\vec{k}$$

$$(b) \vec{r}''(t) = 4e^{-2t}\vec{i} - 4\sin 2t\vec{j} + 2\vec{k}$$

5.8

$$\text{곡률} : \kappa = 1, \quad \text{곡률반경} : \rho = 1$$

5.9

$$\text{곡률} : \kappa = 1$$

$$\text{단위법선벡터} : \vec{N} = \cos 2t\vec{i} - \sin 2t\vec{j}$$

5.10

$$(a) \vec{r}'(t) = (2t-2)\vec{i} + (-2te^{-2t} + e^{-2t})\vec{j}$$

$$\vec{r}''(t) = 2\vec{i} + (4te^{-2t} - 4e^{-2t})\vec{j}$$

$$(b) \vec{r}'(t) = 4t\vec{i} - \frac{2}{(t-1)^2}\vec{j}$$

$$\vec{r}''(t) = 4\vec{i} + \frac{4}{(t-1)^3}\vec{j}$$

$$(c) \vec{r}'(t) = (2te^{-t} - t^2e^{-t})\vec{i} + (2\sin t + 2t\cos t - e^t)\vec{j} + (-2e^{-2t}\cos t - e^{-2t}\sin t)\vec{k}$$

$$\vec{r}''(t) = (2e^{-t} - 4te^{-t} + t^2e^{-t})\vec{i} + (4\cos t - 2t\sin t - e^t)\vec{j} + (3e^{-2t}\cos t + 4e^{-2t}\sin t)\vec{k}$$

5.11

$$(a) z_x = 2x\cos y + 15x^2y^2$$

$$z_y = -x^2\sin y + 10x^3y + 6y^2$$

$$(b) \quad z_x = 6x^2y + 2xy^2 + 2y$$

$$z_y = 2x^3 + 2x^2y + 2x - 3y^2$$

$$(c) \quad z_x = \frac{2xy + 2}{y^2 + 1}$$

$$z_y = \frac{2xy^2 + 2x - 2x^2y^2 - 4xy}{(y^2 + 1)^2}$$

$$(d) \quad z_x = 9\cos^2 2x(-2\sin 2x) - 2x \sin^3 2y$$

$$z_y = -3x^2 \sin^2 2y (2\cos 2x)$$

5.12

$$(a) \quad z_{xx} = 4\sin y - 4y^3$$

$$z_{yy} = -2x^2 \sin y - 12x^2 y$$

$$z_{xy} = 4x \cos y - 12xy^2 - 2$$

$$(b) \quad z_{xx} = -12x^2 y^2$$

$$z_{yy} = -2x^4 - 6xy$$

$$z_{xy} = 1 - 8x^3 y - 3y^2$$

$$(c) \quad z_{xx} = 8y^2 \cos 4x$$

$$z_{yy} = 16x \cos 4y - 2\cos^2 2x$$

$$z_{xy} = 4\sin 4y (y + 1)$$

$$(d) \quad z_{xx} = 4(x^2 - xy + 2y^2) + 2(2x - y)^2 + 2$$

$$z_{yy} = 8(x^2 - xy + 2y^2) + 2(4y - x)^2 + 2$$

$$z_{xy} = -2(x^2 - xy + 2y^2) + 2(4y - x)(2x - y)$$

5.13

$$(a) \quad \frac{\partial z}{\partial u} = (2x - 2xy^2)(2uv - e^{-v}) + (-2x^2y + 2y)(4uv - 2u)$$

$$\frac{\partial z}{\partial v} = (2x - 2xy^2)(u^2 + ue^{-v}) + (-2x^2y + 2y)(2u^2)$$

$$(b) \quad \frac{\partial z}{\partial u} = (4xy - y^2)(-2u + 4uv) + (2x^2 - 2xy)(6u^2v + 2uv^2 + 2v)$$

$$\frac{\partial z}{\partial v} = (4xy - y^2)(2u^2 - 3v^2) + (2x^2 - 2xy)(2u^3 + 2u^2v + 2u)$$

5.14

$$(a) \frac{dz}{dt} = (2x - 2xy^2)(4\cos 2t - 4\sin 2t) + 2x^2y(2\sin t + \cos t)$$

$$(b) \frac{dz}{dt} = (-ye^{-x} - e^{-y})(e^{-2t}\sin t - 3e^{-2t}\cos t) + (e^{-x} + xe^{-y})(4t - 1)$$

5.15

$$(a) \nabla f(1, 2) = -4\vec{i} - 8\vec{j}$$

$$(b) \nabla f(1, 2, 1) = 8\vec{i} + 7\vec{j}$$

5.16

$$(a) D_{\vec{a}}f = 4\sqrt{2}$$

$$(b) D_{\vec{a}}f = -\sqrt{3}$$

5.17

$$\nabla H(5, 1) = -20\vec{i} - 6\vec{j}$$

5.18

온도가 가장 급격하게 증가하는 방향 : $4\vec{i} - 2\vec{k}$

증가하는 최댓값 : $2\sqrt{5}$

온도가 가장 급격하게 감소하는 방향 : $-4\vec{i} + 2\vec{k}$

감소하는 최솟값 : $-2\sqrt{5}$

5.19

$$(a) \operatorname{div} \vec{F}(x, y, z) = 3$$

$$(b) \operatorname{div} \vec{F}(x, y, z) = e^y + xy$$

$$(c) \operatorname{div} \vec{F}(x, y, z) = e^{-z} + e^{-x} + e^{-y}$$

5.20

생략

5.21

(a) $\text{curl } \vec{F} = \vec{0}$

(b) $\text{curl } \vec{F} = (xz - e^{-x})\vec{i} - yz\vec{j} + (e^{-x}z + xe^y)\vec{k}$

(c) $\text{curl } \vec{F} = -ze^{-y}\vec{i} - xe^{-z}\vec{j} - ye^{-x}\vec{k}$

5.22

속도장 $\vec{v}$ 는 비회전적이다.