

CHAPTER 09

9.1

$$(a) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$(b) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$(c) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$(d) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$(e) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ -1 & -1 & -1 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$(f) \quad \frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ -3 & -2 & -1 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

9.2

$$(a) \quad \frac{dx}{dt} = -x + 2y, \quad \frac{dy}{dt} = x - 2y$$

$$(b) \quad \frac{dx}{dt} = x - 2y, \quad \frac{dy}{dt} = 3x - 2y$$

$$(c) \quad \frac{dx}{dt} = -x + 2y + z, \quad \frac{dy}{dt} = x - 2y + 2z, \quad \frac{dz}{dt} = 3y - z$$

$$(d) \quad \frac{dx}{dt} = -x + 2y, \quad \frac{dy}{dt} = 2x - y - 2z, \quad \frac{dz}{dt} = -x + 3y + 3z$$

9.3

생략

9.4

$$(a) \quad x = -c_1 + c_2 e^{-3t}, \quad y = c_1 + 2c_2 e^{-3t}$$

$$(b) \quad x = c_1 e^t - c_2 e^{-t}, \quad y = -3c_1 e^t + c_2 e^{-t}$$

$$(c) \quad x = c_1 e^{4t} + c_2 t e^{4t}, \quad y = -\frac{1}{2} c_2 e^{4t}$$

$$(d) \quad x = -c_2 e^{3t}, \quad y = (c_1 + c_2 t) e^{3t}$$

$$\begin{aligned} \text{(e)} \quad x &= C_1 e^{2t} \sin t + C_2 e^{2t} \cos t \\ y &= (C_1 + C_2) e^{2t} \sin t + (C_2 - C_1) e^{2t} \cos t \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad x &= -3c_1 e^{-2t} + c_3 e^{2t} \\ y &= 2c_1 e^{-2t} - c_2 e^t + 2c_3 e^{2t} \\ z &= c_1 e^{-2t} + c_2 e^t + c_3 e^{2t} \end{aligned}$$

$$\begin{aligned} \text{(g)} \quad x &= -c_2 e^{2t} \\ y &= c_1 e^t + 3c_2 e^{2t} + c_3 e^{3t} \\ z &= c_1 e^t + c_2 e^{2t} \end{aligned}$$

$$\begin{aligned} \text{(h)} \quad x &= c_1 e^{5t} - c_2 e^{2t} - c_3 e^{2t} \\ y &= c_1 e^{5t} + c_2 e^{2t} \\ z &= c_1 e^{5t} + c_3 e^{2t} \end{aligned}$$

$$\begin{aligned} \text{(i)} \quad x &= c_1 e^t + c_2 e^{2t} - c_3 e^{2t} \\ y &= c_1 e^t + c_2 e^{2t} \\ z &= c_1 e^t + c_3 e^{2t} \end{aligned}$$

$$\begin{aligned} \text{(j)} \quad x &= c_3 e^t \\ y &= c_2 e^t + c_3 t e^t - 2c_3 e^t \\ z &= c_1 e^t + c_2 t e^t + c_3 \frac{t^2}{2} e^t \end{aligned}$$

$$\begin{aligned} \text{(k)} \quad x &= c_1 e^t + c_2 t e^t + c_2 e^t + c_3 \frac{t^2}{2} e^t + c_3 t e^t \\ y &= c_3 \frac{1}{2} e^t \\ z &= c_1 e^t + c_2 t e^t + c_3 \frac{t^2}{2} e^t \end{aligned}$$

9.5

$$\text{(a)} \quad \vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4c_1 e^t - 2c_2 e^{-t} + 2c_3 e^{3t} \\ -4c_1 e^t \\ c_1 e^t + c_2 e^{-t} + c_3 e^{3t} \end{bmatrix}$$

$$\text{(b)} \quad \vec{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -c_1 e^{5t} - c_2 e^{5t} + c_3 e^{8t} \\ c_1 e^{5t} + c_3 e^{8t} \\ c_2 e^{5t} + c_3 e^{8t} \end{bmatrix}$$

9.6

$$(a) \quad x = -c_1 e^t - c_2 e^{-t} + 5, \quad y = 3c_1 e^t + c_2 e^{-t} - 8$$

$$(b) \quad \vec{X} = \begin{bmatrix} x \\ y \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{6t} + \begin{bmatrix} -\frac{13}{18} \\ \frac{8}{9} \end{bmatrix}$$

$$(c) \quad \vec{X} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{4t} + \begin{bmatrix} -\frac{9}{8} \\ \frac{11}{8} \end{bmatrix}$$

$$(d) \quad \vec{X} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^t + \begin{bmatrix} 2t-3 \\ -3t+4 \end{bmatrix}$$

$$(e) \quad x = (-c_1 \sin t + c_2 \cos t) e^t - \frac{1}{2}t - \frac{1}{2}$$

$$y = (c_1 \cos t + c_2 \sin t) e^t + \frac{1}{2}t$$

$$(f) \quad \vec{X} = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{5t} + c_2 \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} t e^{5t} + \begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix} e^{5t} \right\} + \begin{bmatrix} -\frac{3}{5} + \frac{1}{18} e^{-t} \\ \frac{1}{6} e^{-t} \end{bmatrix}$$

9.7

9.6의 결과와 같다. (생략)

9.8

$$(a) \quad \vec{X} = c_1 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} e^t + \begin{bmatrix} \frac{23}{9} - \frac{4}{3}t \\ 0 \\ -\frac{11}{9} + \frac{1}{3}t \end{bmatrix}$$

$$(b) \quad \vec{X} = c_1 \begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ \frac{3}{2} \\ -\frac{5}{2} \end{bmatrix} e^{5t} + c_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} e^{7t} + \begin{bmatrix} \frac{2}{5} + \frac{1}{4} e^{3t} \\ -\frac{2}{5} - \frac{3}{16} e^{3t} \\ \frac{5}{16} e^{3t} \end{bmatrix}$$

9.9

$$(a) \quad x = -c_1 e^{-t} + c_2 e^{4t} - \frac{5}{6} e^t$$

$$y = c_1 e^{-t} + \frac{3}{2} c_2 e^{4t} + \frac{1}{2} e^t$$

$$\begin{aligned} \text{(b)} \quad x &= -c_1 e^t - c_2 e^{-t} + 2t - 3 \\ y &= 3c_1 e^t + c_2 e^{-t} - 3t + 4 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad x &= c_1 e^{3t} + c_2 e^{-t} - \frac{2}{3} + \frac{2}{5} e^{4t} \\ y &= c_1 e^{3t} - c_2 e^{-t} + \frac{4}{3} + \frac{3}{5} e^{4t} \\ z &= c_3 e^{2t} - \frac{1}{4} - \frac{1}{2} t \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad x &= -c_1 e^{-2t} - 2c_2 e^{-2t} + c_3 e^t + \frac{3}{10} e^t + 2 \\ y &= c_2 e^{-2t} + c_3 e^t + \frac{1}{10} e^{3t} \\ z &= c_1 e^{-2t} + c_2 e^{-2t} + c_3 e^t + \frac{1}{10} e^{3t} + 1 \end{aligned}$$

9.10

$$\begin{aligned} \text{(a)} \quad i_1 &= 10t + c_1 \\ i_2 &= i_1 - i_3 \\ i_3 &= c_2 e^{-t} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad i_1 &= -\frac{1}{2} c_1 e^{-2t} + c_2 + 5t \\ i_2 &= \frac{1}{2} c_1 e^{-2t} + c_2 + 5t - 5 \\ i_3 &= i_1 - i_2 \end{aligned}$$